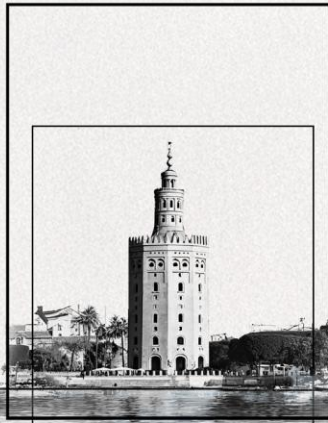


20-24 JULIO 2026  
SEVILLA



Escuela Técnica Superior de Ingeniería, Universidad de Sevilla

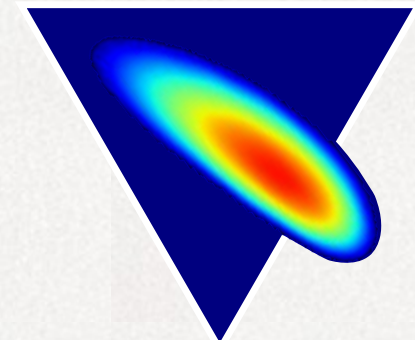
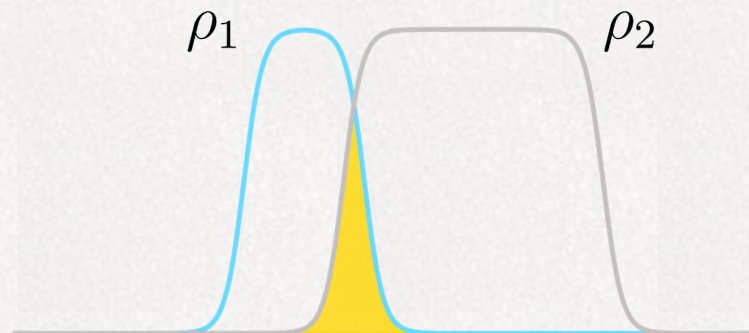


# A NOVEL OPTICAL MODEL PROTOCOL FOR NUCLEAR REACTION ANALYSES AT NEAR-BARRIER ENERGIES

*L. Garrido Gómez, J. P. Fernández García, M. A. G. Alvarez*

Dpto. de Física Atómica, Molecular y Nuclear  
Universidad de Sevilla

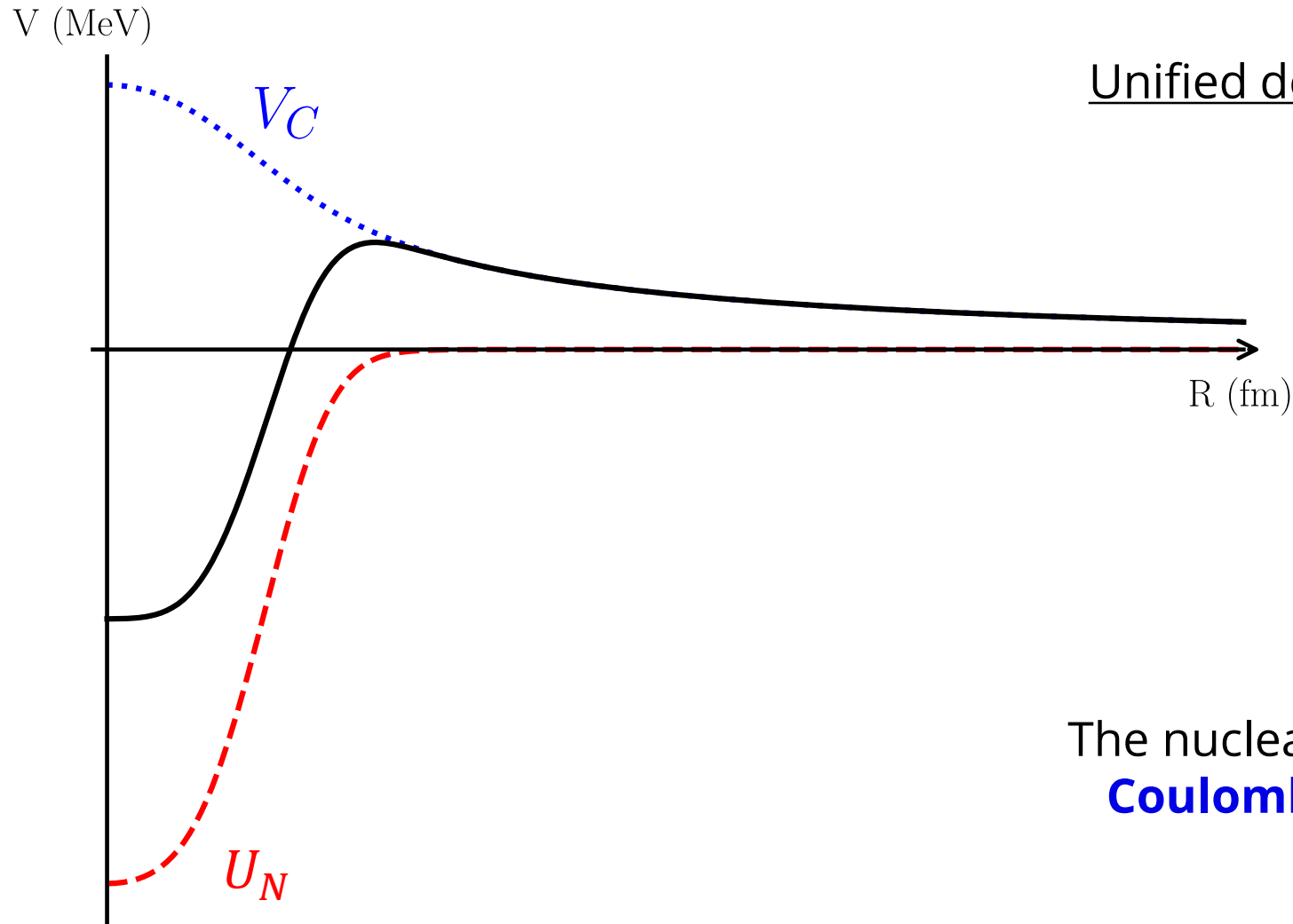
$$V(R) = U_0 e^{-4v^2/c^2} \iint \rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2) e^{-(r_{12}/a)^2} d\mathbf{r}_1 d\mathbf{r}_2$$



# Contents

- Motivations
- The São Paulo Optical Model Protocol (SP-OMP)
- What can we learn from the SP-OMP
- Application to  $X + {}^{64}\text{Zn}$  reactions
- New measurements of  ${}^{12,13}\text{C} + {}^{64}\text{Zn}$

# Motivations

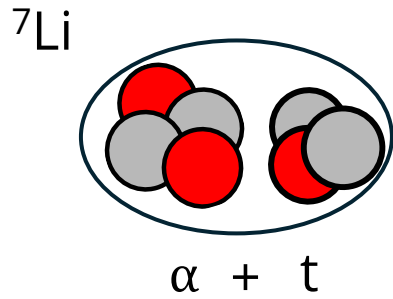
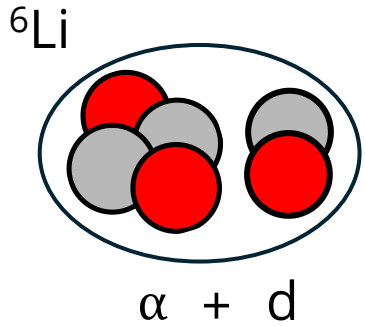


Unified description of the nuclear interaction

$$V_C + \boxed{U_N} \quad ? \quad ?$$

The nuclear potential must include the **Coulomb** and **nuclear** interactions

# Motivations

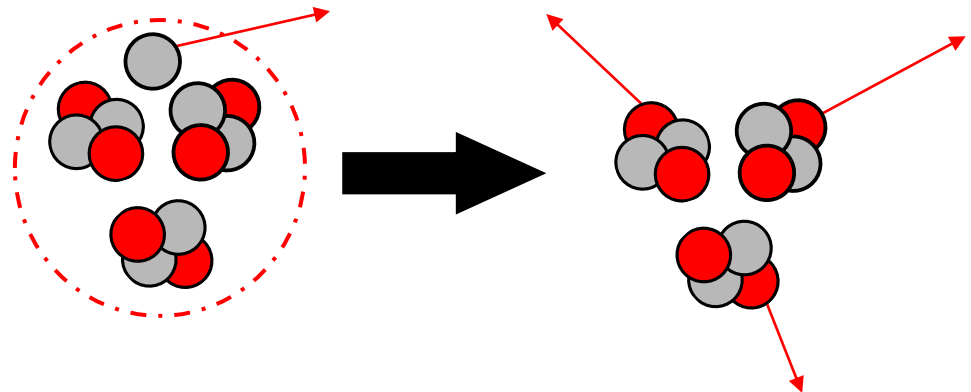
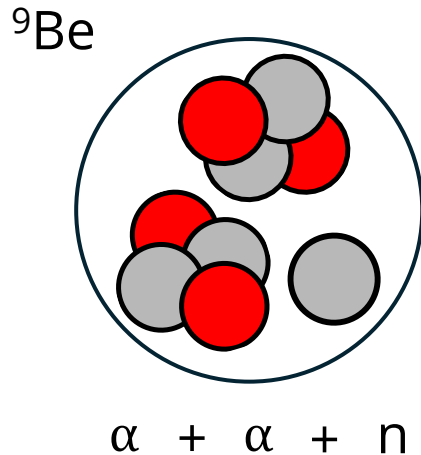
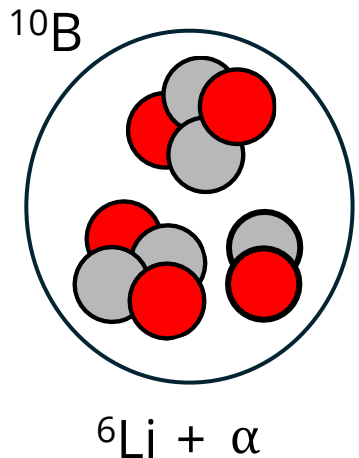


Structure of matter and mechanisms of nuclear reactions

**Cluster structures**



How does nuclear structure affect  
nuclear reaction dynamics?



# The São Paulo Optical Model Protocol

In the **Optical Model formalism** the nuclear potential is **complex**

$$V_C + U_N$$

The diagram shows the equation  $V_C + U_N$  at the top. A red circle highlights  $U_N$ . Two red arrows point from  $U_N$  down to the equation  $U_N(R) = V(R) + iW(R)$ . From this equation, a red arrow points down to a yellow box describing the real part, and another red arrow points right to a yellow box describing the imaginary part.

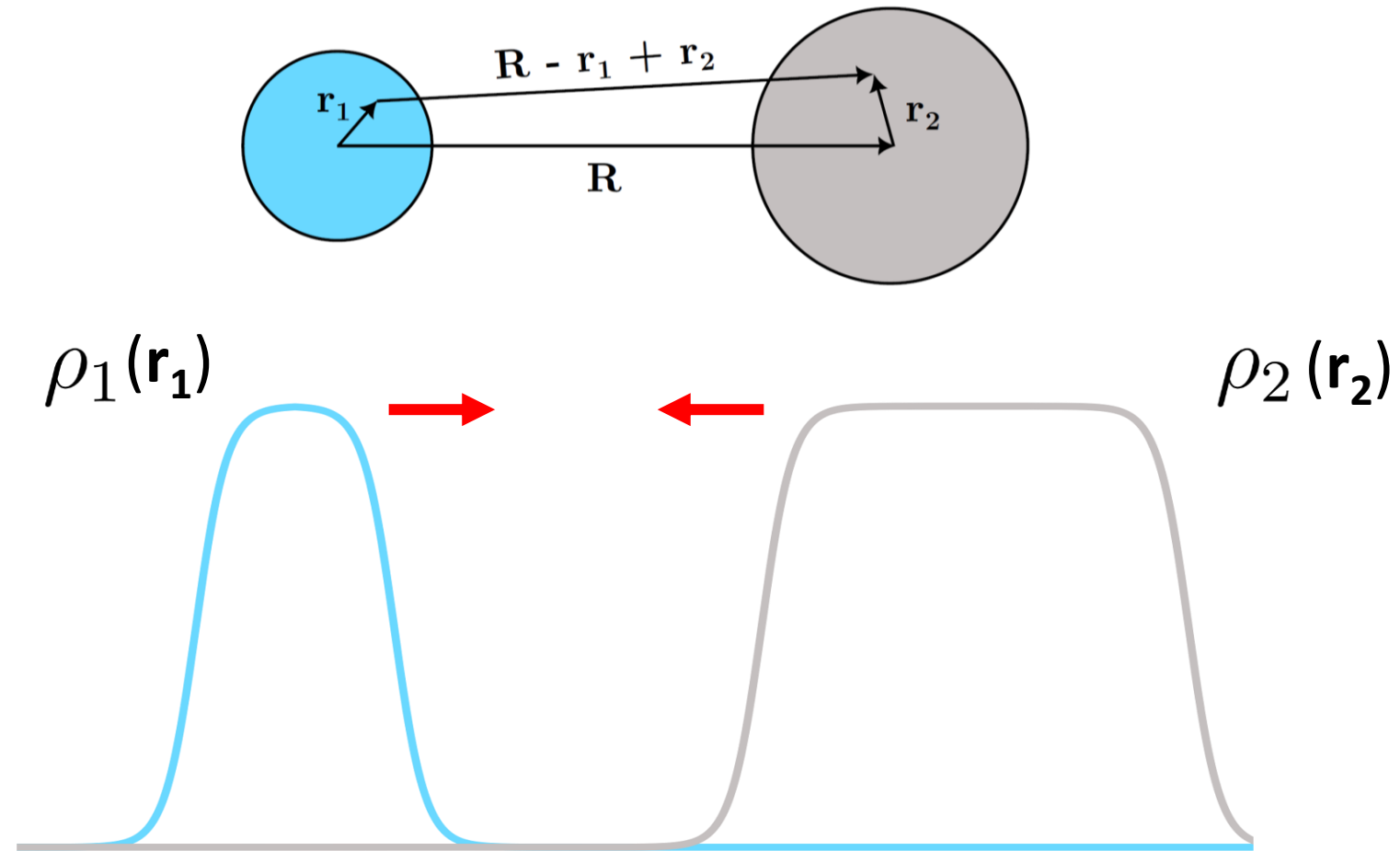
$$U_N(R) = V(R) + iW(R)$$

The **imaginary part** accounts for the **non-elastic processes**

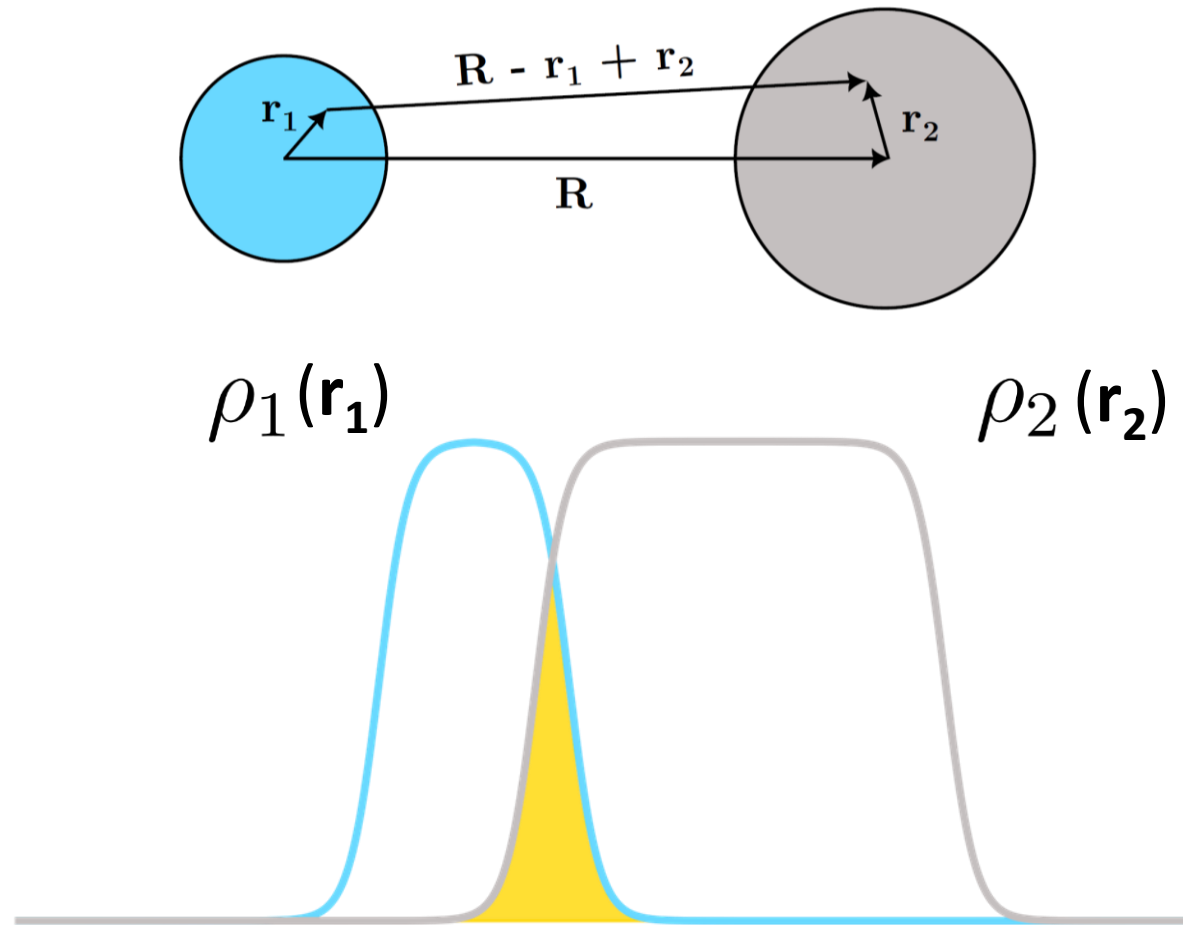
(absorption from the elastic flux)

The **real part** models the **nuclear interaction** itself

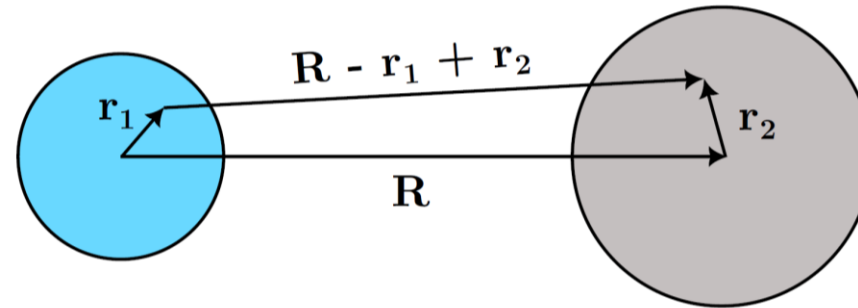
# The São Paulo Optical Model Protocol



# The São Paulo Optical Model Protocol



# The São Paulo Optical Model Protocol



$$V(R) = \iint \underbrace{\rho_1(\mathbf{r}_1)}_{\text{matter densities}} \underbrace{\rho_2(\mathbf{r}_2) \nu(\mathbf{R} - \mathbf{r}_1 + \mathbf{r}_2)}_{\text{effective nucleon-nucleon interaction}} d\mathbf{r}_1 d\mathbf{r}_2$$

matter densities

effective nucleon-nucleon interaction

# The São Paulo Potential 2 (SPP2)

$$V(R) = U_0 e^{-4v^2/c^2} \iint \underbrace{\rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2)}_{\text{Theoretical distributions (Dirac-Hartree-Bogoliubov)}} e^{-(r_{12}/a)^2} d\mathbf{r}_1 d\mathbf{r}_2$$

$\downarrow$

$U_0 = -735.813 \text{ MeV}$

$\downarrow$

$a = 0.50 \text{ fm}$

**Theoretical** distributions (Dirac-Hartree-Bogoliubov)

**Experimental** distributions (electron scattering experiments)

**No FREE parameters!**

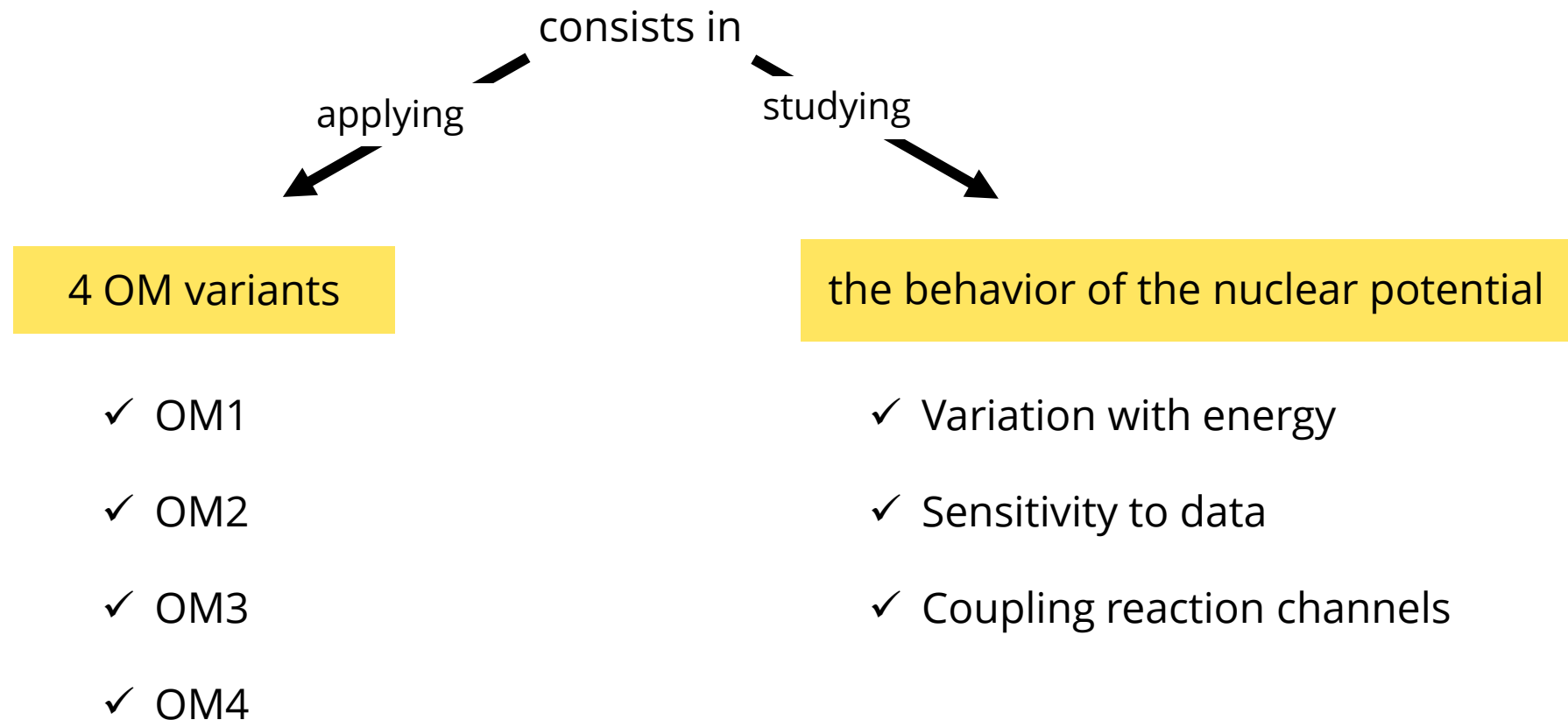
*B. V. Carlson et al., Phys. Rev. C 62, 054310 (2000)*

*L. C. Chamon et al., Comput. Phys. Commun. 267, 108061 (2021)*

# The São Paulo Optical Model Protocol

## The SP-OMP

(São Paulo Optical Model Protocol)



# The São Paulo Optical Model Protocol (SP-OMP)

**OM1**

$$U_N(R) = V_{\text{SPP2}}(R) + iW(R)$$

This **internal potential** simulates the **fusion process** and does not include other peripheral reactions

The **real term** uses the **SPP2** parameterization

The **imaginary term** uses the **WS** parameterization

OM1 has **no free parameters**

$$W(R) = \frac{W_0}{1 + \exp\left(\frac{R-R_0}{a}\right)}$$

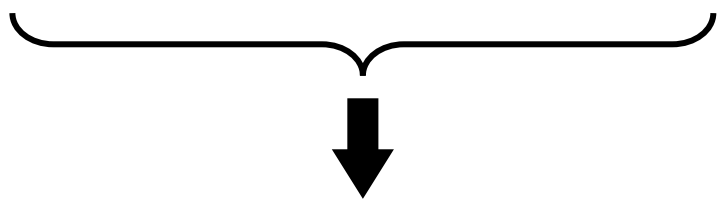
$$\begin{cases} W_0 = -100 \text{ MeV} \\ R_0 = r_0(A_1^{1/3} + A_2^{1/3}) \\ r_0 = 1.06 \text{ fm} \\ a = 0.25 \text{ fm} \end{cases}$$

# The São Paulo Optical Model Protocol (SP-OMP)

## OM2

This **superficial potential** simulates peripheral reactions

OM2 has **no free parameters**

$$U_N(R) = N_r V_{\text{SPP2}} + iN_i V_{\text{SPP2}}$$


Both **real** and **imaginary** parts use the **SPP2** parameterization

The **standard** parameters:

$$N_r = 1$$

$$N_i = 0.78$$

# The São Paulo Optical Model Protocol (SP-OMP)

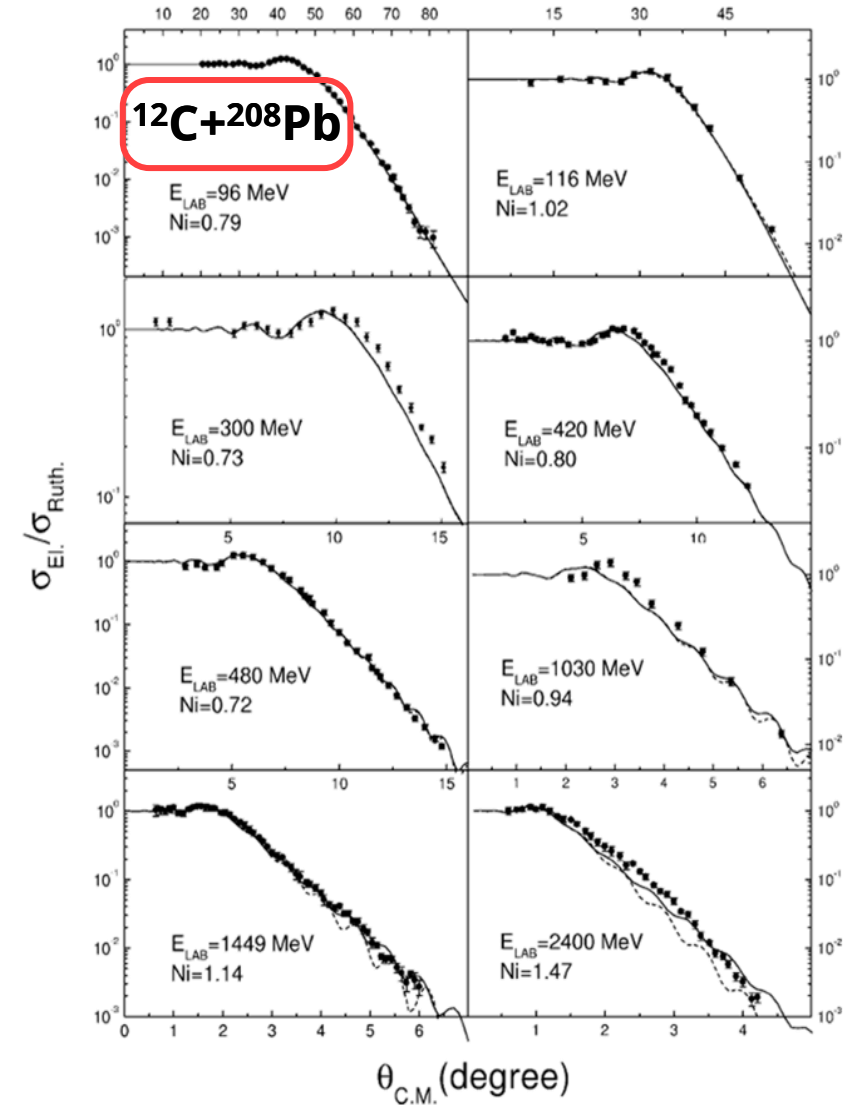
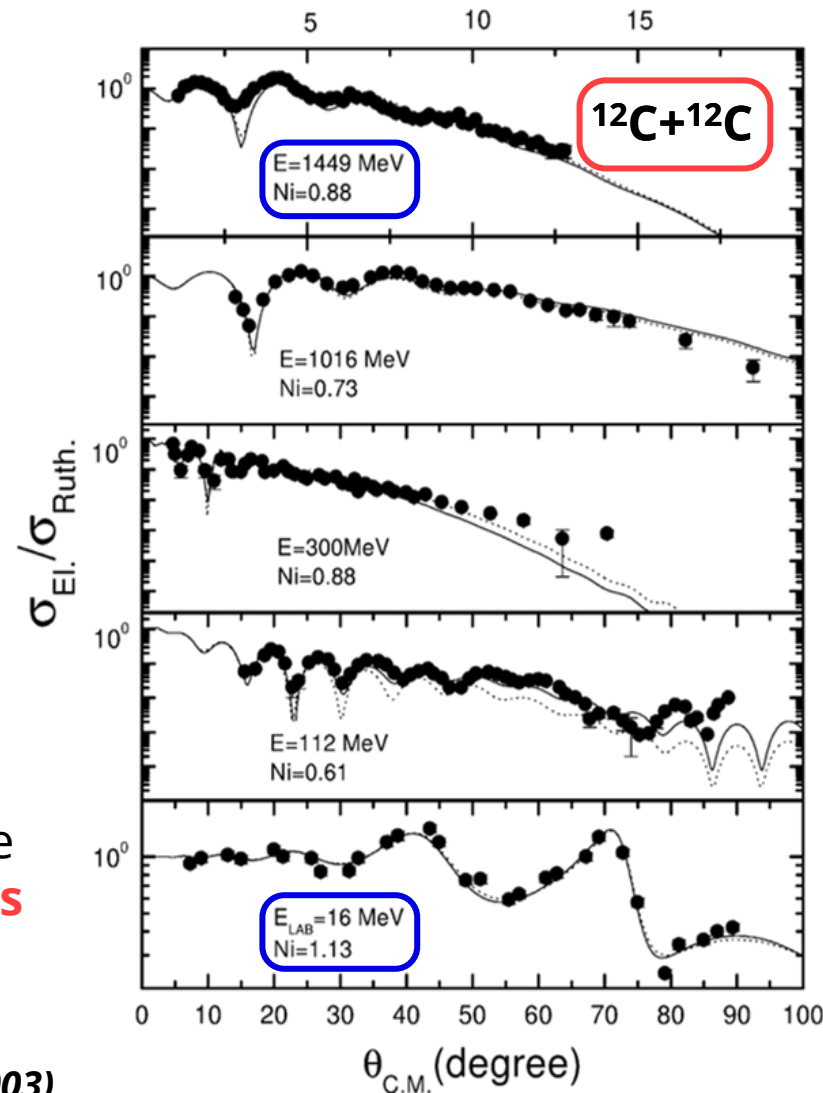
OM2

The **standard** parameters:

$$N_r = 1$$

$$N_i = 0.78$$

Good description in a wide range of **energies** and **reduced masses**



M. A. G. Alvarez et al. *Nucl. Phys. A* 723, 93 (2003)

# The São Paulo Optical Model Protocol (SP-OMP)

**OM3**

$$U_N(R) = N_r V_{\text{SPP2}} + i \boxed{N_i} V_{\text{SPP2}}$$

The **imaginary strength** is allowed to vary in order to **fit** the data

Both **real** and **imaginary** parts use the **SPP2** parameterization

Simulates **opening/closing** reaction channels

OM3 is a **one-parameter fit**

# The São Paulo Optical Model Protocol (SP-OMP)

**OM4**

$$U_N(R) = \underbrace{N_r V_{\text{SPP2}} + i N_i V_{\text{SPP2}}}$$

The **real and imaginary strengths** are allowed to vary in order to **fit** the data

Both **real** and **imaginary** parts use the **SPP2** parameterization

Simulates **opening/closing** reaction channels and **polarization effects**

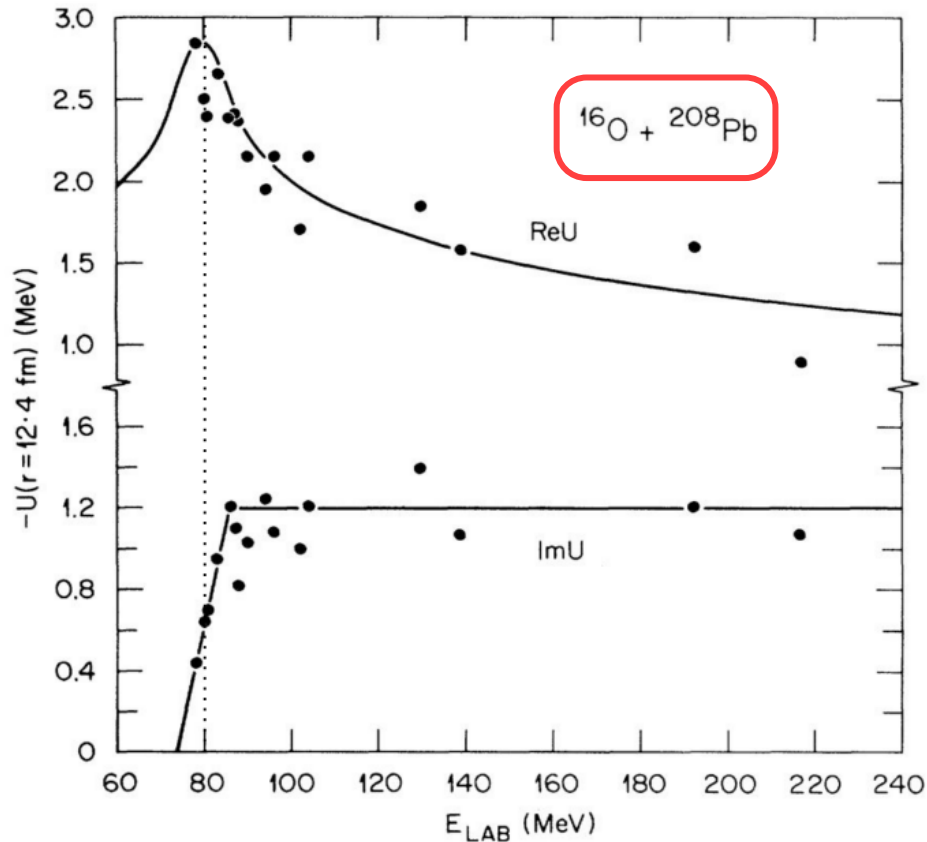
OM4 is a **two-parameter fit**

# The São Paulo Optical Model Protocol (SP-OMP)

## The dispersion relation

OM4 allows us to study the relationship between the real and imaginary strengths

*M. A. Nagarajan et al., Phys. Rev. Lett. 54, 1136 (1985)*



$$U_N(R) = N_r V_{\text{SPP2}} + iN_i V_{\text{SPP2}}$$

Real strength increases below the barrier

Imaginary strength vanishes below the barrier

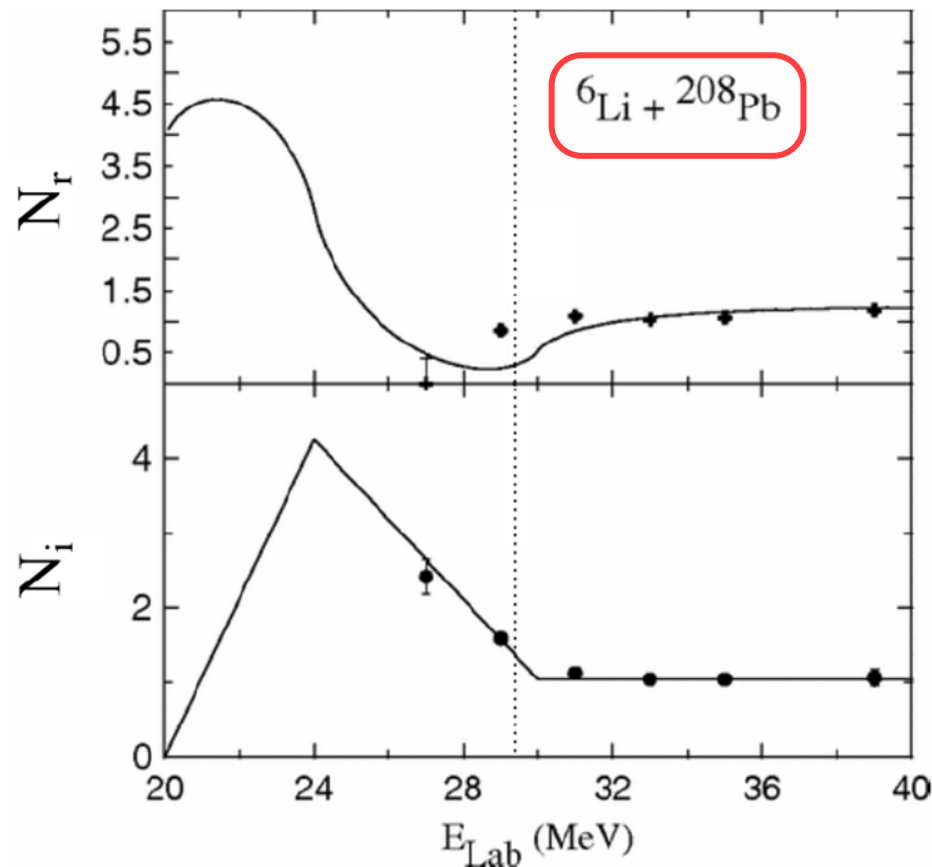
Threshold anomaly

# The São Paulo Optical Model Protocol (SP-OMP)

## The dispersion relation

OM4 allows us to study the relationship between the real and imaginary strengths

*M. S. Hussein et al., Phys. Rev. C 76, 019902 (2007)*



$$U_N(R) = N_r V_{\text{SPP2}} + iN_i V_{\text{SPP2}}$$

Real strength decreases

Breakup threshold anomaly

Significant imaginary strength below the barrier

# Application to He, Li, Be, B, C projectiles at near-barrier energies

Application to  $^4,^6\text{He}$ ,  $^6,^7\text{Li}$ ,  $^8\text{B}$ ,  $^9,^{10},^{11}\text{Be}$  on a  $^{64}\text{Zn}$  target



Data from:

- A. Di Pietro *et al.*, **Phys. Rev. C** **69**, 044613 (2004)
- M. Zadro *et al.*, **Phys. Rev. C** **80**, 064610 (2009)
- A. Di Pietro *et al.*, **Phys. Rev. C** **85**, 054607 (2012)
- J. P. Fernández-García *et al.*, **Phys. Rev. C** **92**, 054602 (2015)
- J. P. Fernández-García *et al.*, **Phys. Rev. C** **99**, 054605 (2019)
- R. Spartà *et al.*, **Phys. Lett. B** **820**, 136477 (2021)

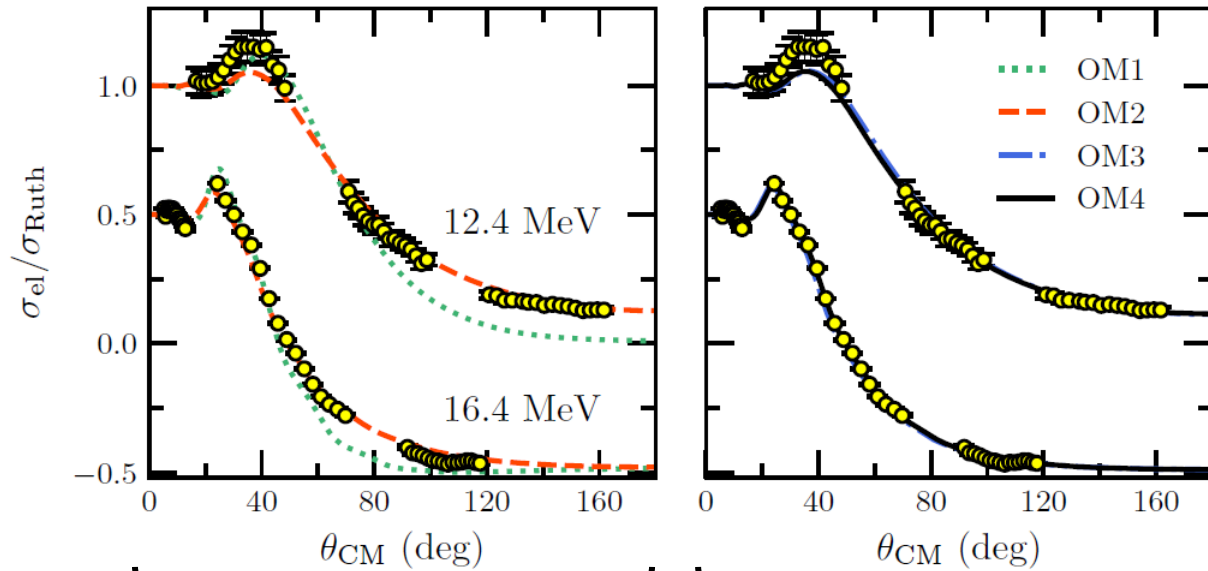


All calculations are performed using **FRESCO** and the **SPP2**

We study the **OP strengths** as a function of the **projectile** and the **bombarding energy**

# Application to $^4\text{He}$ , $^6\text{Li}$ , $^8\text{B}$ , $^9,^{10},^{11}\text{Be}$ on a $^{64}\text{Zn}$ target

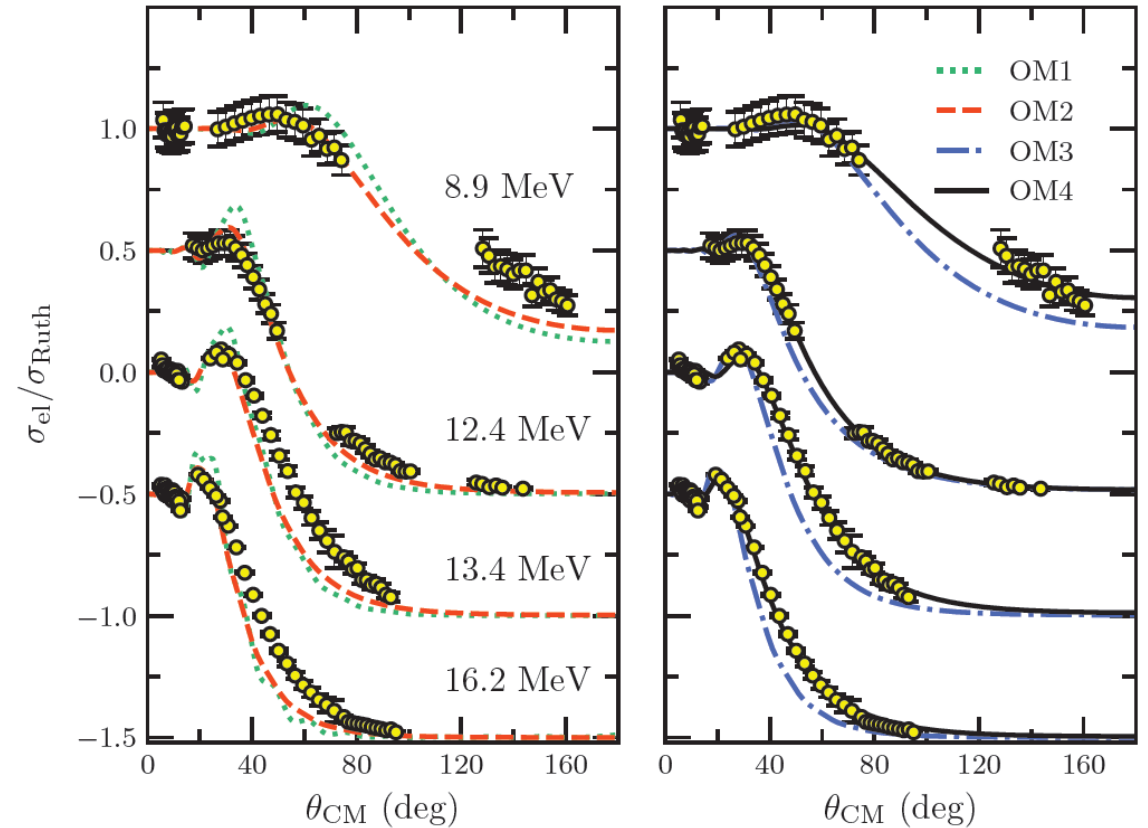
## $^4\text{He} + ^{64}\text{Zn}$



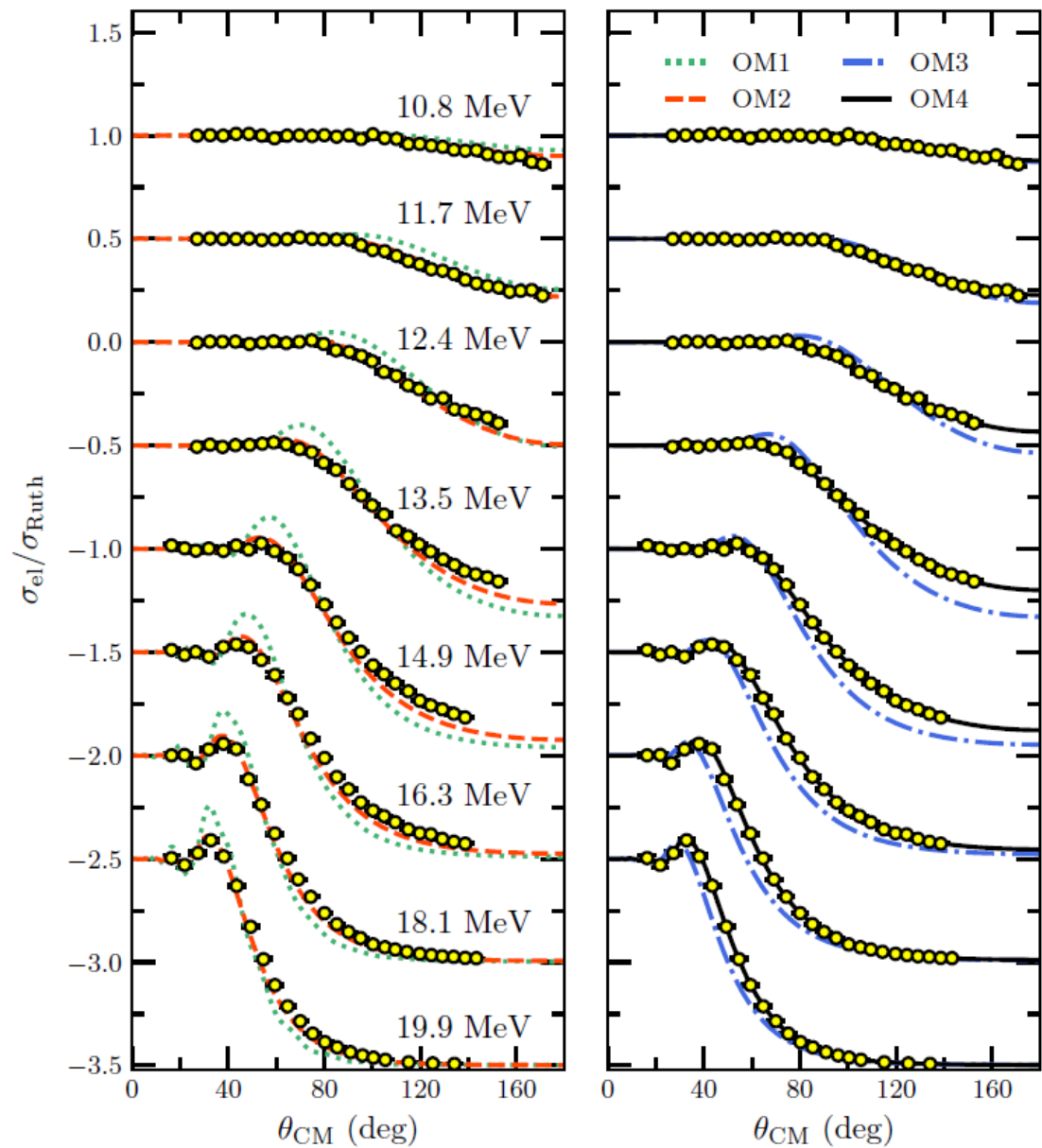
OM1 and OM2:  
Parameter free

OM3 and OM4:  
Parameter fit

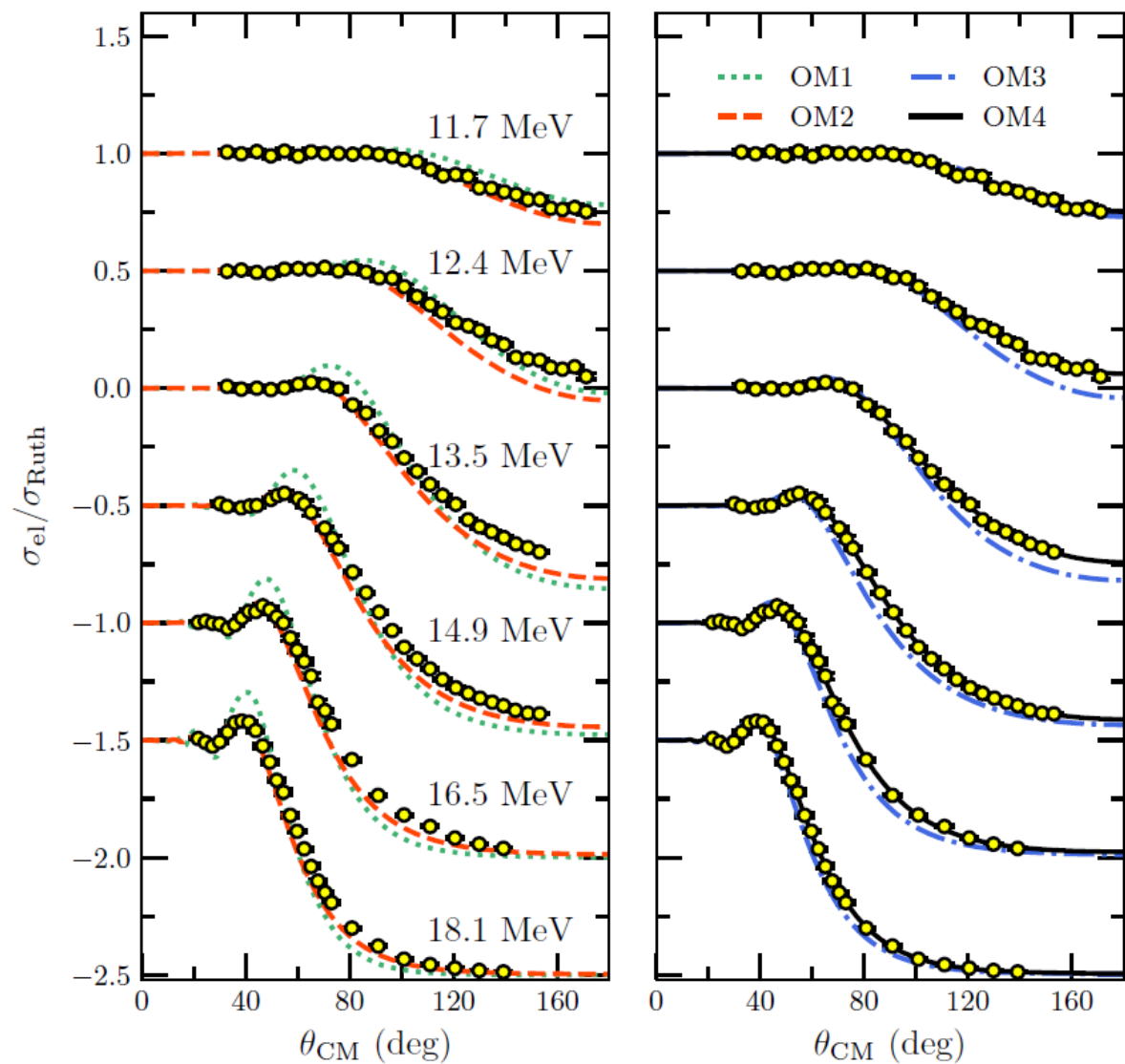
## $^6\text{He} + ^{64}\text{Zn}$



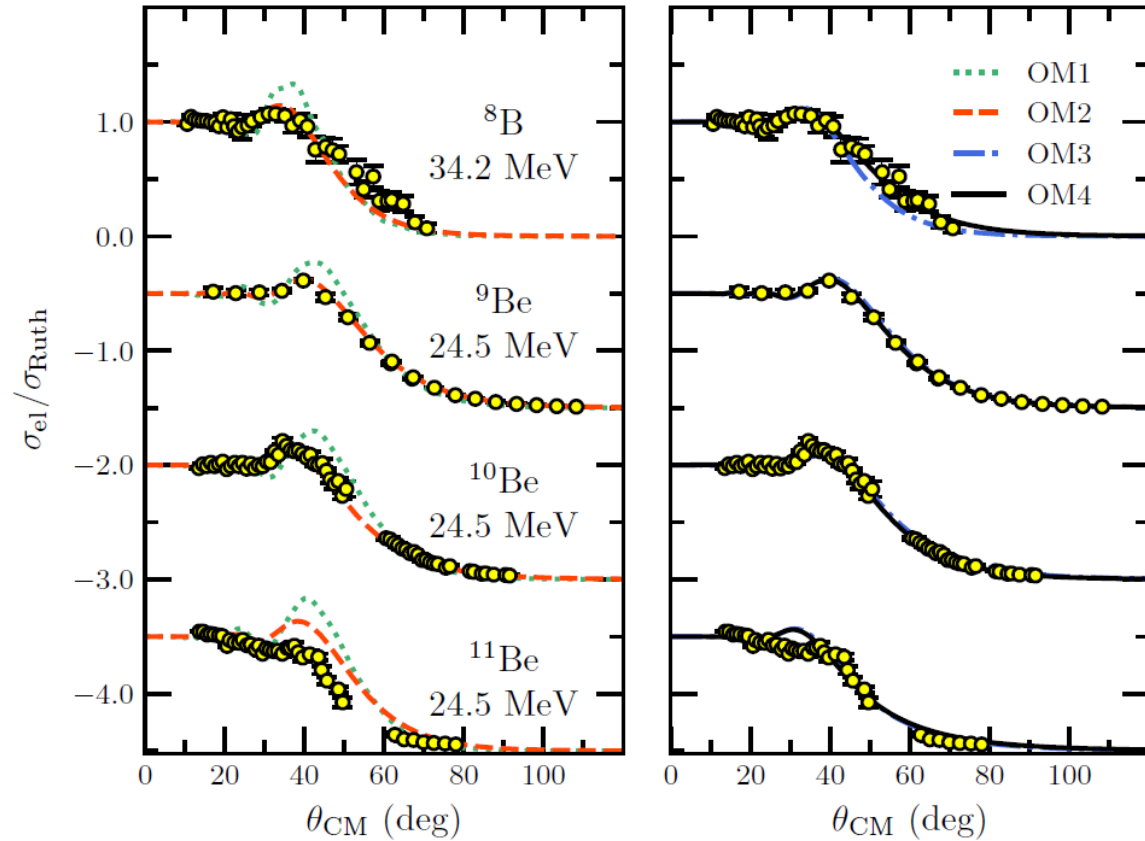
# ${}^6\text{Li} + {}^{64}\text{Zn}$



# ${}^7\text{Li} + {}^{64}\text{Zn}$



# $^8\text{B}, ^{9,10,11}\text{Be} + ^{64}\text{Zn}$

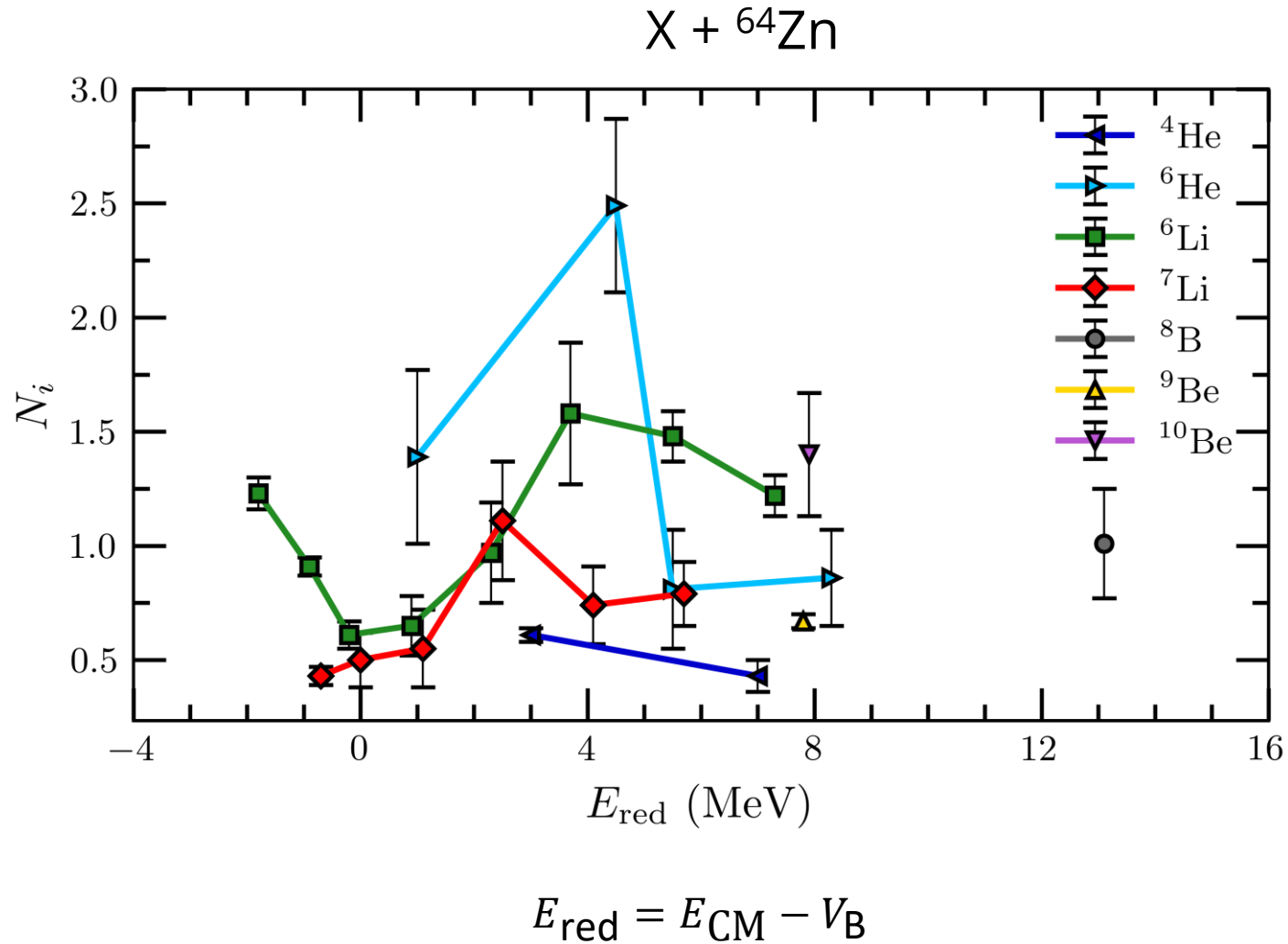


**OM3 and OM4 systematically  
provides the best results**



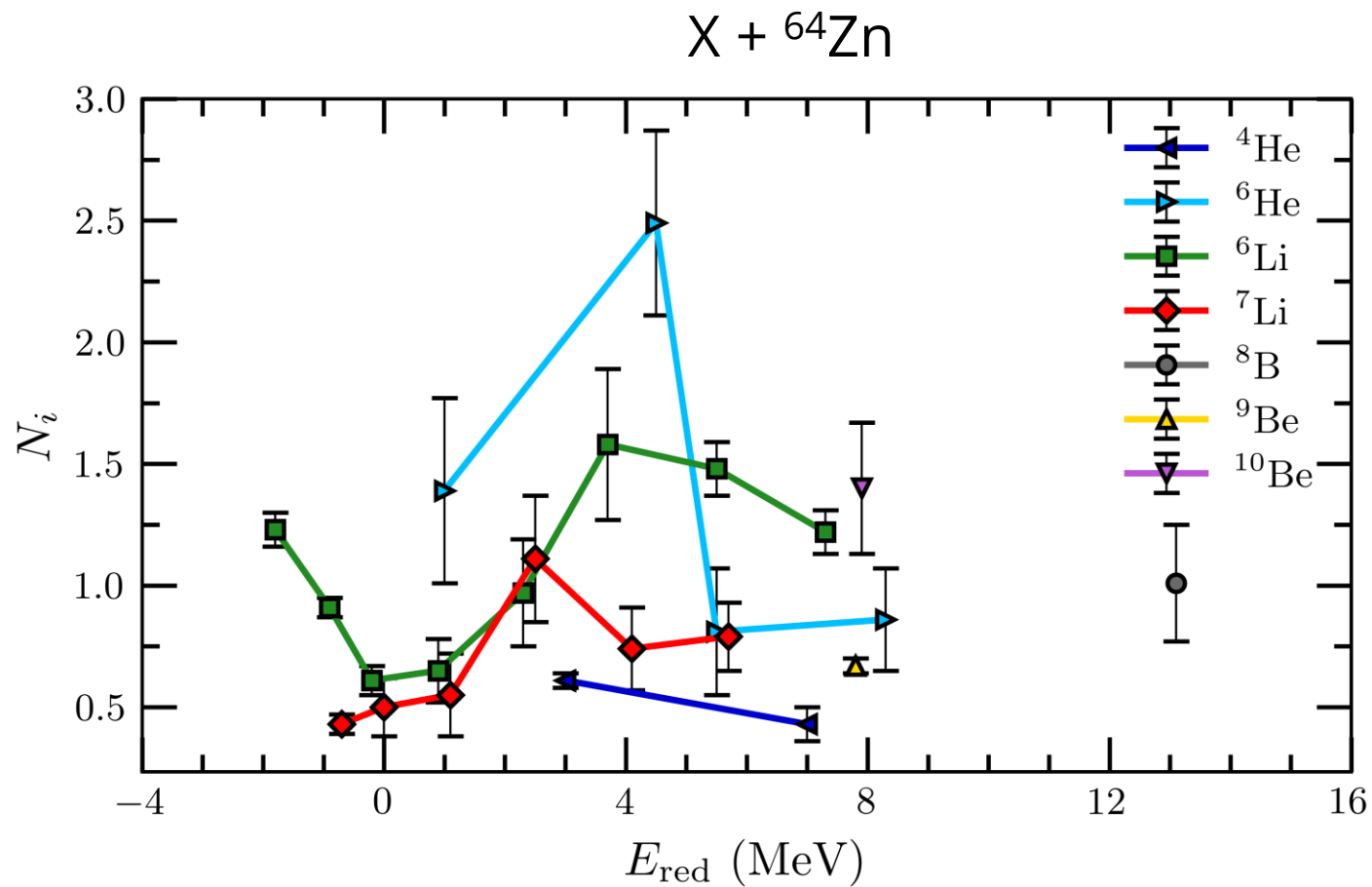
| Projectile       | $V_B$ | $E_{\text{CM}}$ | $\chi^2_{\text{red}}$<br>OM1 | $\chi^2_{\text{red}}$<br>OM2 | $\chi^2_{\text{red}}$<br>OM3 | $\chi^2_{\text{red}}$<br>OM4 |
|------------------|-------|-----------------|------------------------------|------------------------------|------------------------------|------------------------------|
| $^4\text{He}$    | 9.4   | 12.42           | 38                           | 1.44                         | 0.86                         | 0.78                         |
|                  |       | 16.37           | 334                          | 104                          | 68                           | 34                           |
|                  |       |                 |                              |                              |                              |                              |
| $^6\text{He}$    | 7.9   | 8.9             | 4.21                         | 2.34                         | 2.11                         | 0.24                         |
|                  |       | 12.4            | 30                           | 11                           | 3.50                         | 0.39                         |
|                  |       | 13.4            | 130                          | 98                           | 98                           | 66                           |
|                  |       | 16.2            | 585                          | 381                          | 380                          | 196                          |
|                  |       |                 |                              |                              |                              |                              |
| $^6\text{Li}$    | 12.6  | 10.77           | 25                           | 3.88                         | 1.29                         | 1.29                         |
|                  |       | 11.69           | 97                           | 3.69                         | 2.38                         | 0.69                         |
|                  |       | 12.43           | 44                           | 8.25                         | 6.22                         | 0.92                         |
|                  |       | 13.54           | 99                           | 53                           | 51                           | 1.05                         |
|                  |       | 14.92           | 213                          | 101                          | 97                           | 1.15                         |
|                  |       | 16.30           | 341                          | 134                          | 83                           | 2.10                         |
|                  |       | 18.14           | 581                          | 162                          | 42                           | 2.34                         |
| $^7\text{Li}$    | 12.4  | 19.98           | 317                          | 75                           | 29                           | 1.19                         |
|                  |       | 11.69           | 16                           | 7.73                         | 2.30                         | 0.78                         |
|                  |       | 12.43           | 24                           | 74                           | 56                           | 1.24                         |
|                  |       | 13.54           | 129                          | 88                           | 82                           | 0.84                         |
|                  |       | 14.92           | 439                          | 168                          | 154                          | 0.72                         |
|                  |       | 16.46           | 109                          | 47                           | 47                           | 1.06                         |
|                  |       | 18.14           | 144                          | 50                           | 50                           | 0.68                         |
| $^8\text{B}$     | 21.1  | 34.22           | 6.79                         | 2.17                         | 2.08                         | 0.96                         |
| $^9\text{Be}$    | 16.7  | 24.50           | 484                          | 9.17                         | 5.42                         | 3.59                         |
| $^{10}\text{Be}$ | 16.6  | 24.50           | 48                           | 13                           | 11                           | 8.52                         |
| $^{11}\text{Be}$ | 16.2  | 24.50           | 82                           | 47                           | 30                           | 30                           |

# What can we learn from OM3?



**Imaginary strength** as a function of the **energy** and **projectile**

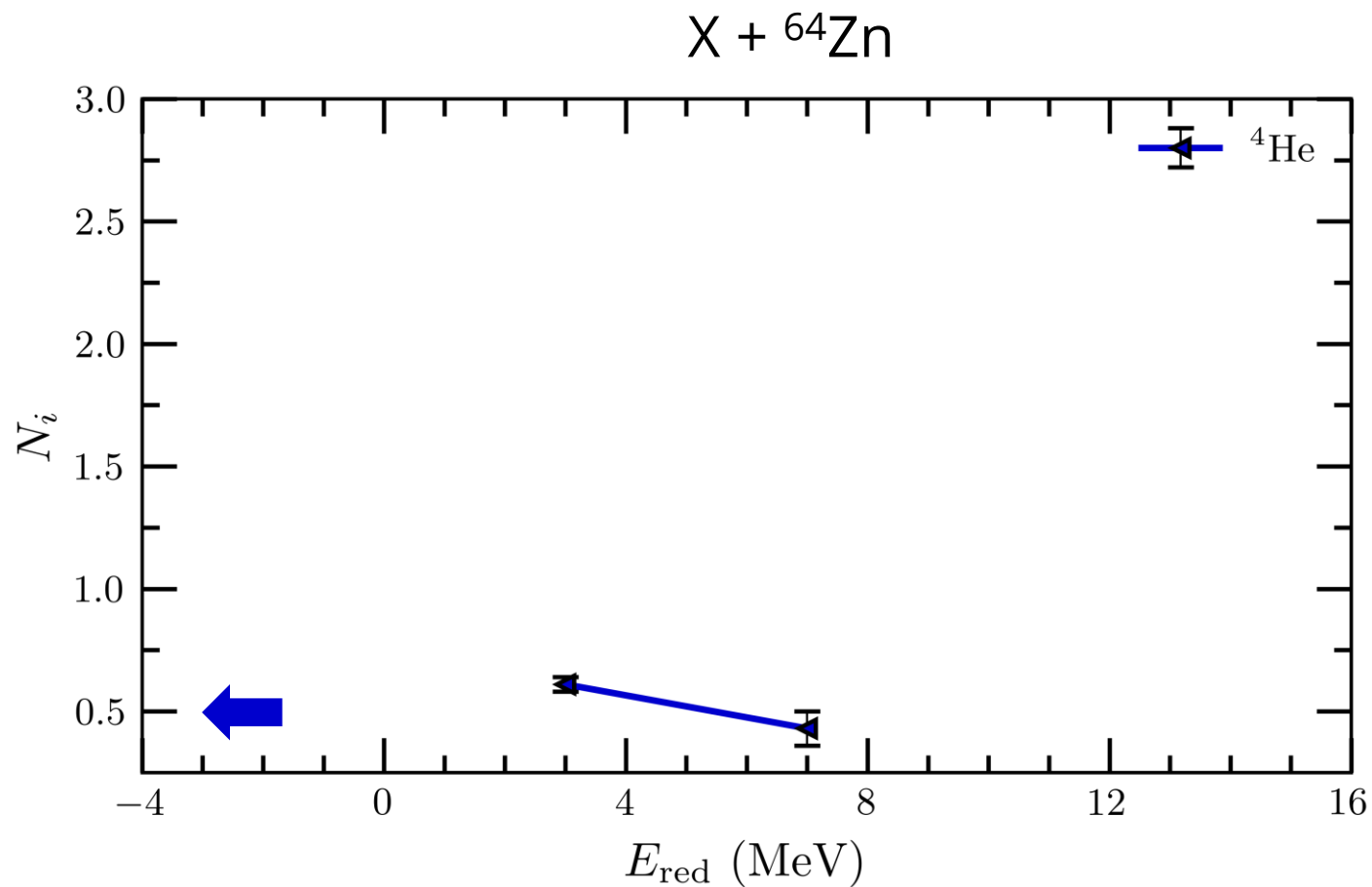
# What can we learn from OM3?



The  $N_i$  values are ordered according to the **projectile binding energy**

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

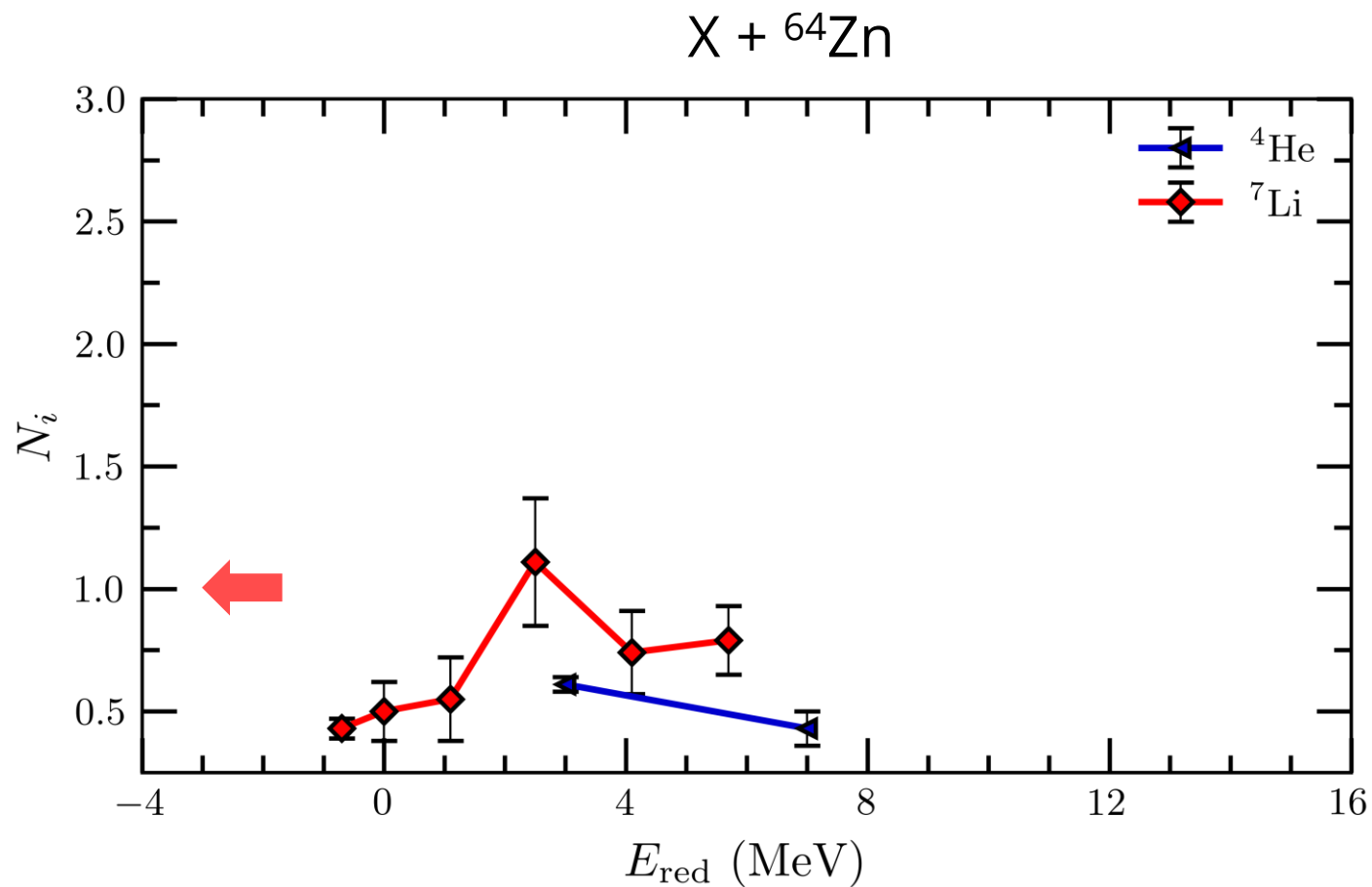
# What can we learn from OM3?



The  $N_i$  values are ordered according to the **projectile binding energy**

| Nucleus            | Cluster                          | $Q_{\text{bu}}$ (MeV) |
|--------------------|----------------------------------|-----------------------|
| ${}^4\text{He}$    | -                                | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + \text{n}$       | -6.812                |
| ${}^7\text{Li}$    | $\text{a} + \text{t}$            | -2.468                |
| ${}^9\text{Be}$    | $\text{a} + \text{a} + \text{n}$ | -1.665                |
| ${}^6\text{Li}$    | $\text{a} + \text{d}$            | -1.474                |
| ${}^6\text{He}$    | $\text{a} + \text{n} + \text{n}$ | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + \text{p}$       | -0.136                |

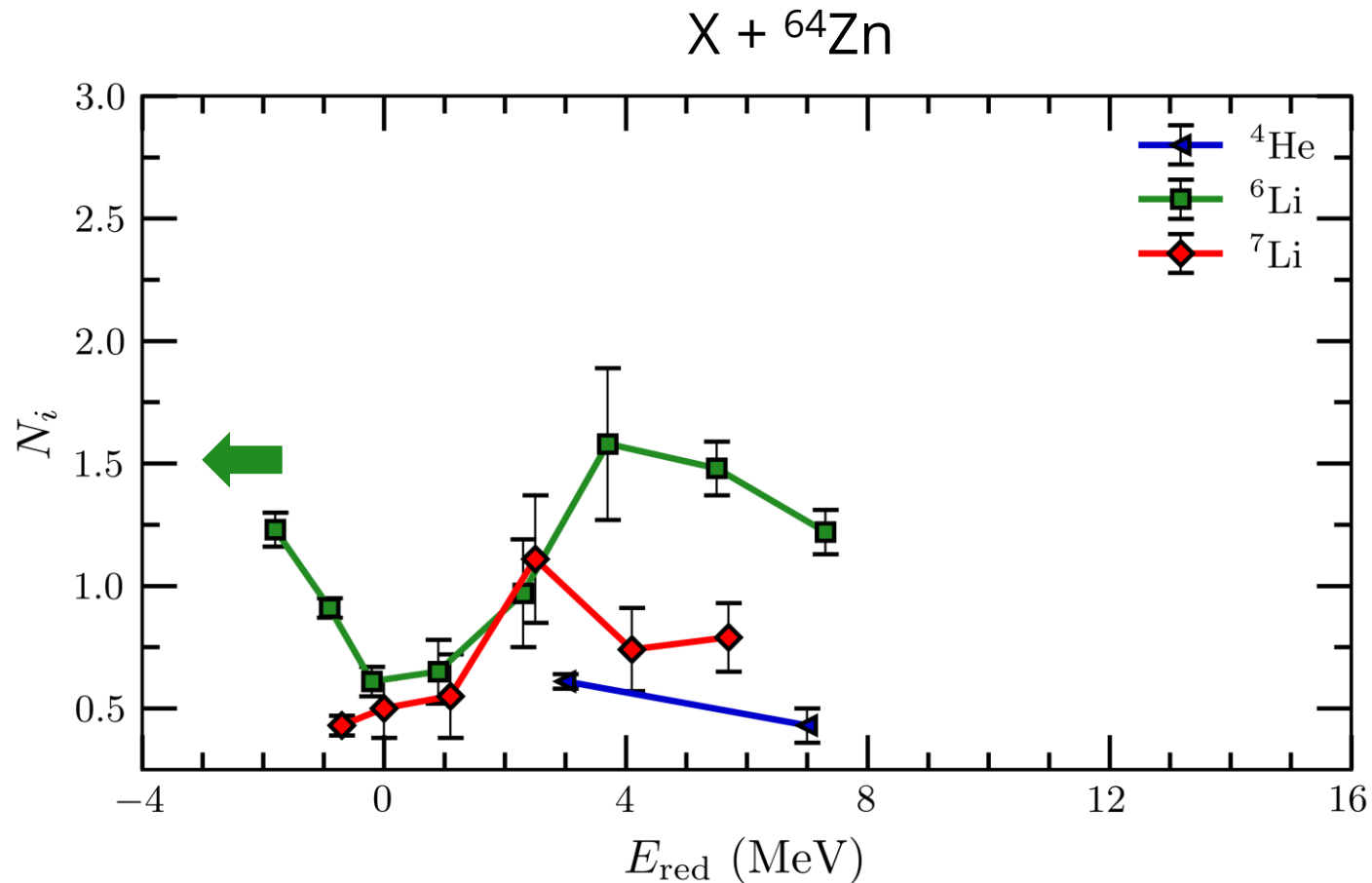
# What can we learn from OM3?



The  $N_i$  values are ordered according to the **projectile binding energy**

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

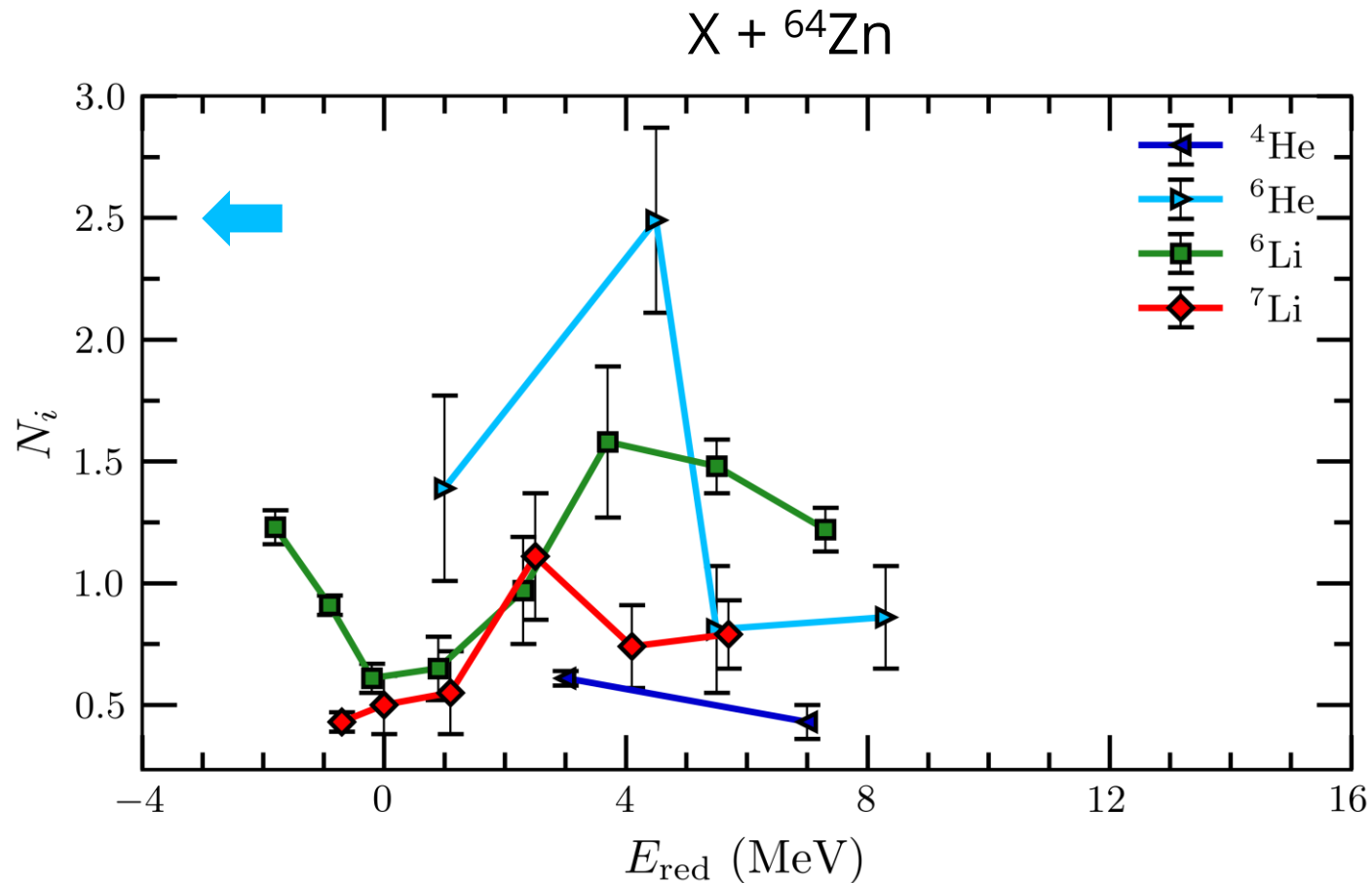
# What can we learn from OM3?



The  $N_i$  values are ordered according to the **projectile binding energy**

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

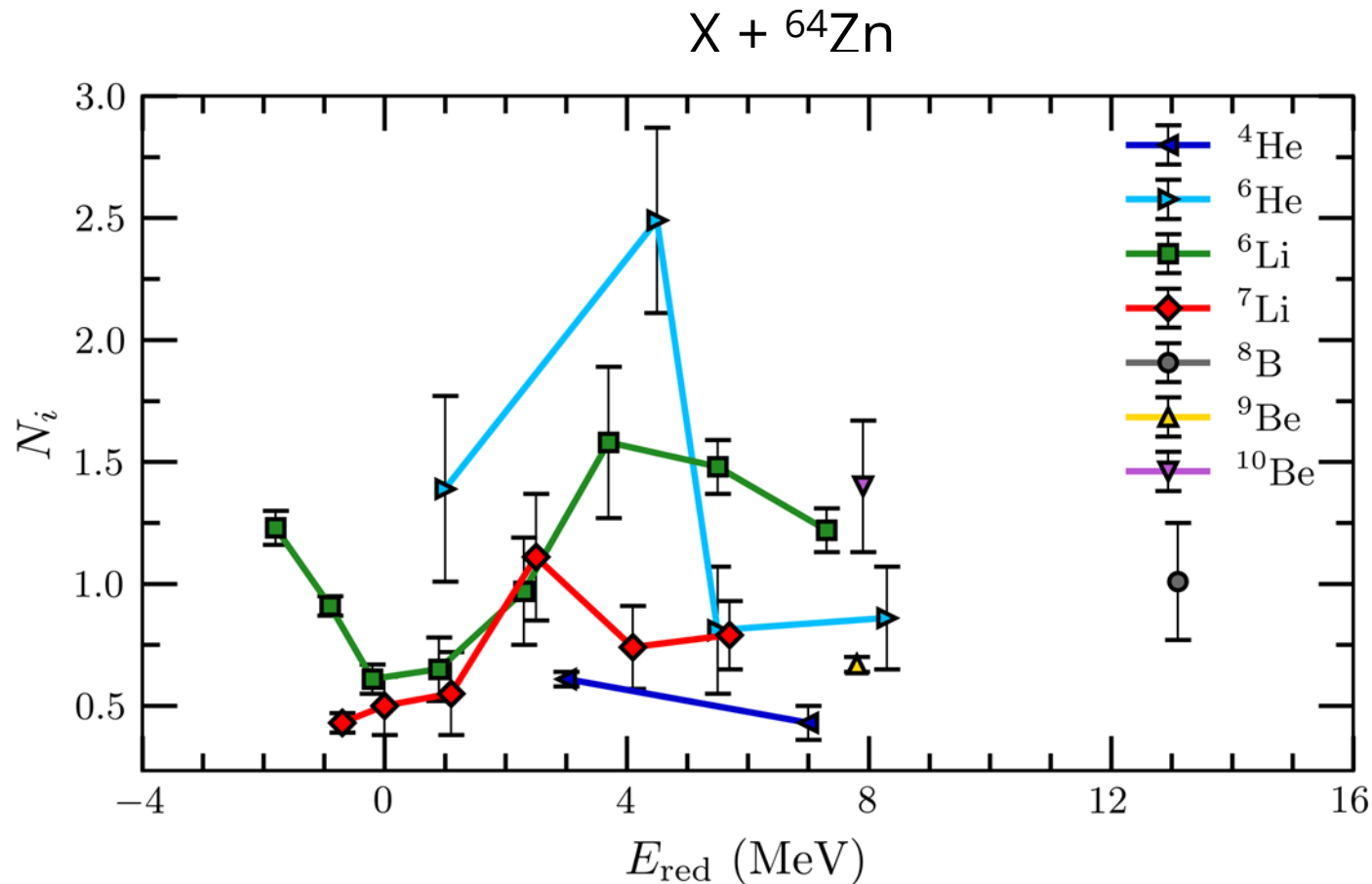
# What can we learn from OM3?



The  $N_i$  values are ordered according to the **projectile binding energy**

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

# What can we learn from OM3?

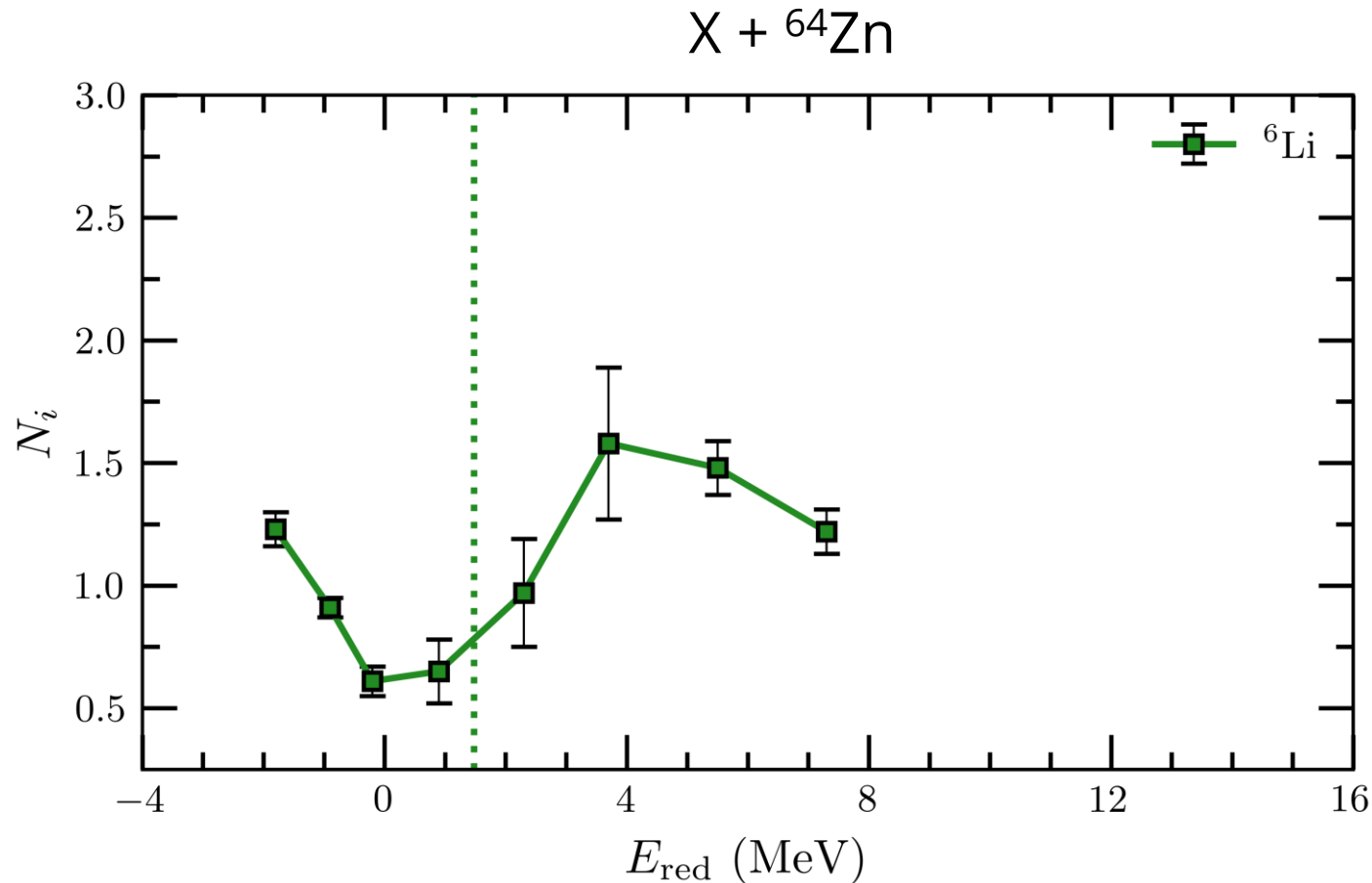


The  $N_i$  values are ordered according to the **projectile binding energy**



**Reaction dynamics** strongly correlates to **projectile structure** for **cluster-structured nuclei**

# What can we learn from OM3?

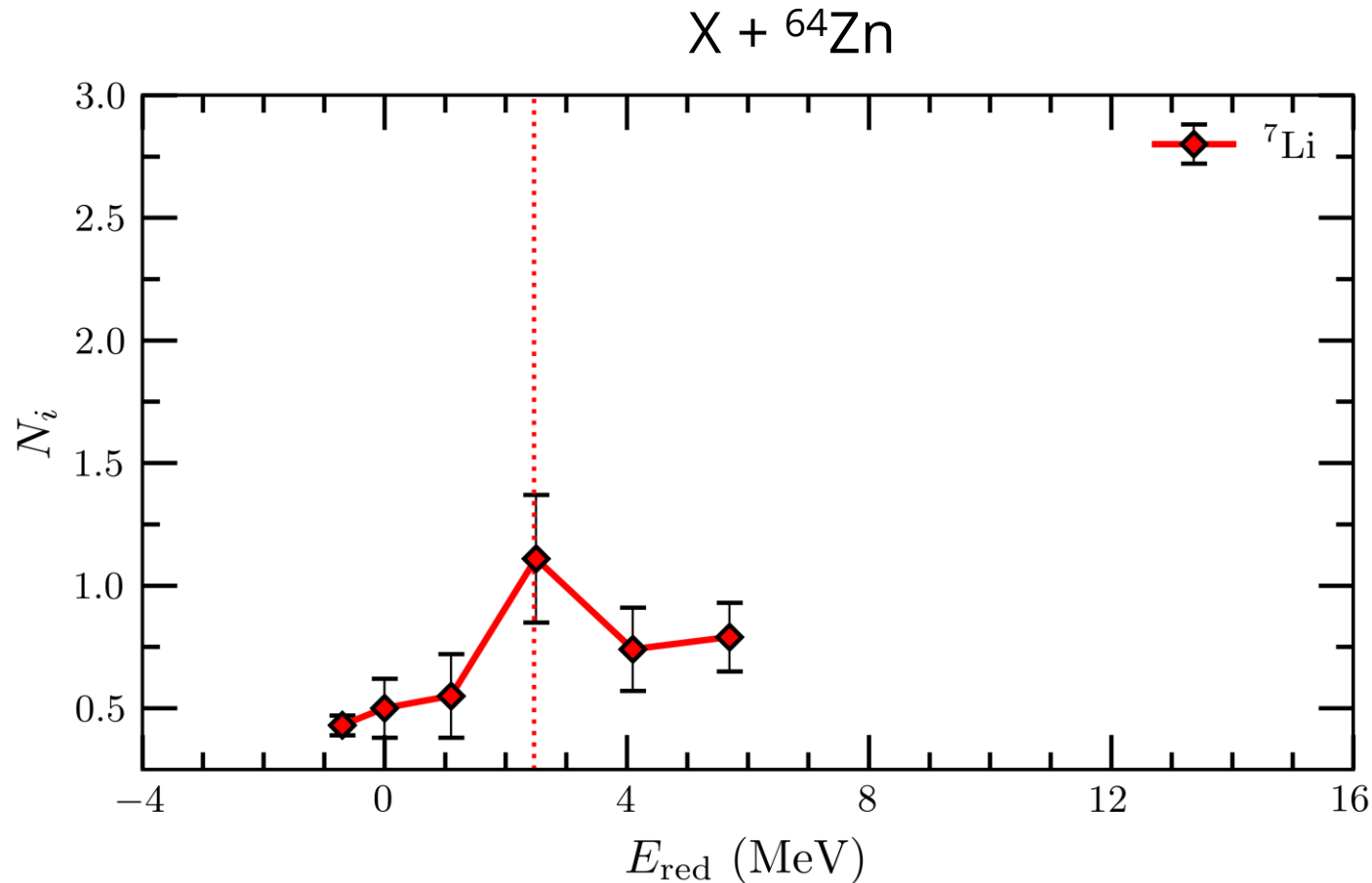


$$E_b^{\text{op}} \geq V_B + |Q_{\text{bu}}|$$

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

The **imaginary strength** presents a **peak structure** at above-barrier energies which correlates with  $Q_{\text{bu}}$

# What can we learn from OM3?

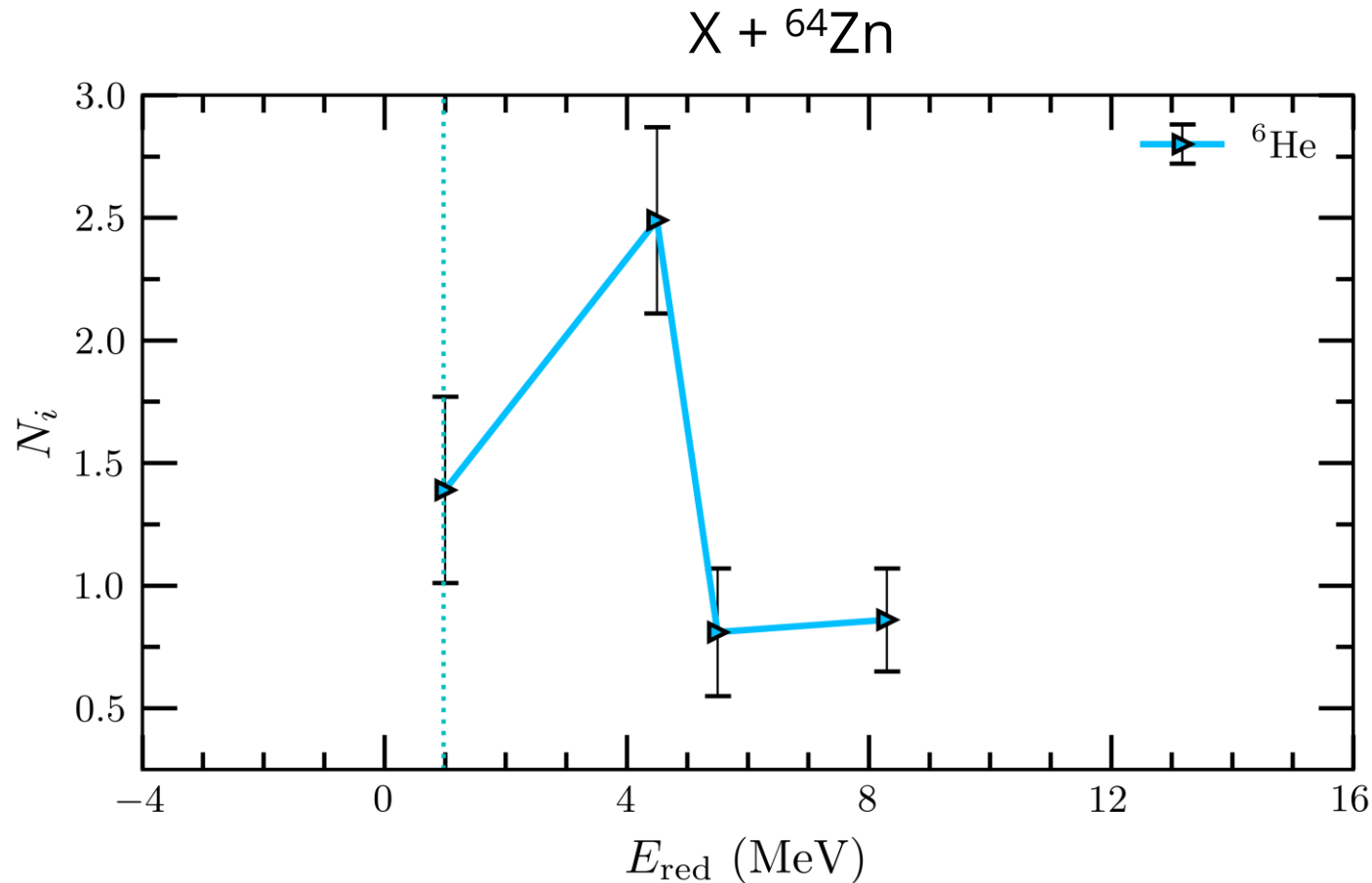


$$E_b^{\text{op}} \geq V_B + |Q_{\text{bu}}|$$

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

The **imaginary strength** presents a **peak structure** at above-barrier energies which correlates with  $Q_{\text{bu}}$

# What can we learn from OM3?

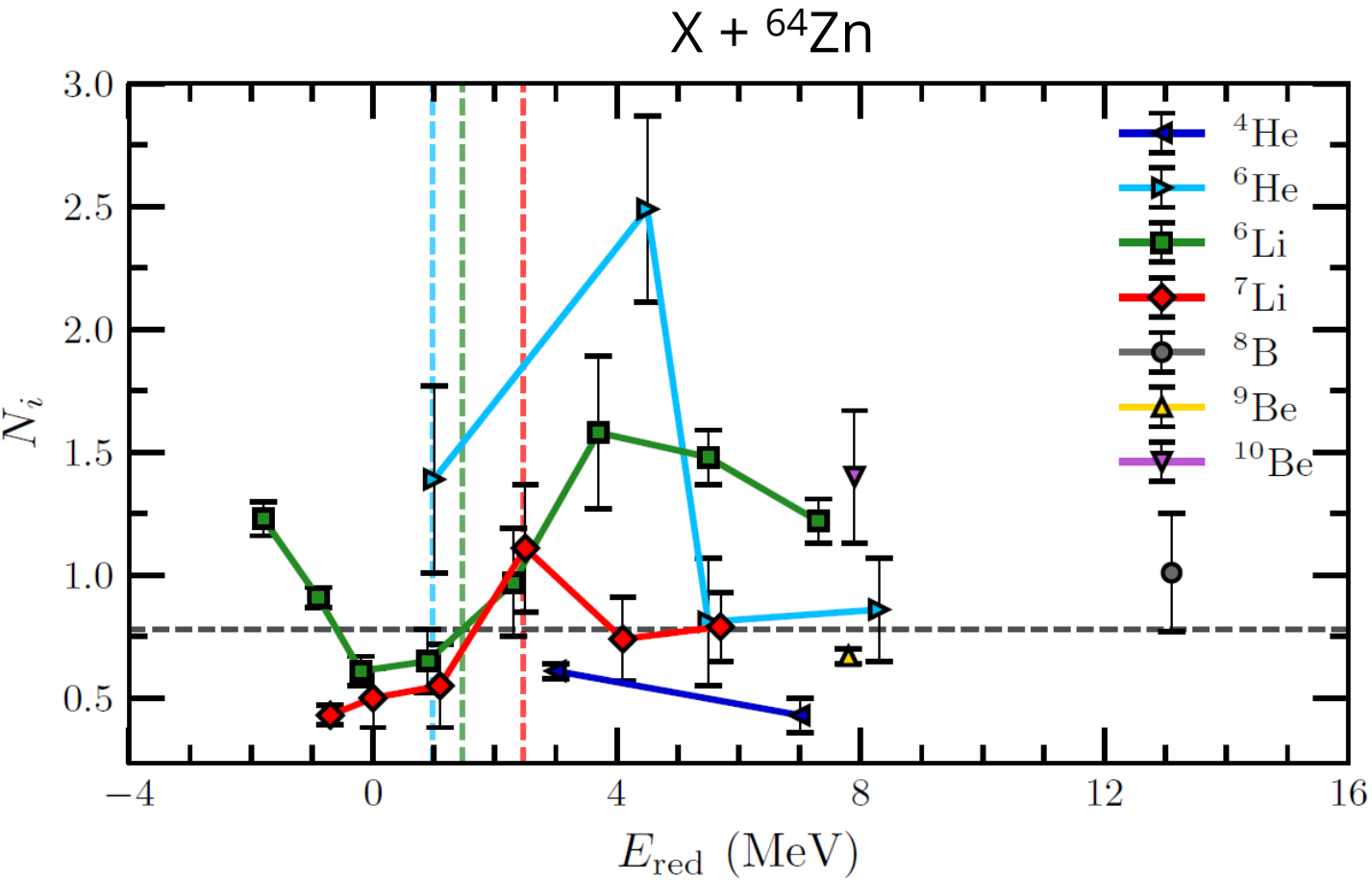


$$E_b^{\text{op}} \geq V_B + |Q_{\text{bu}}|$$

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

The **imaginary strength** presents a **peak structure** at above-barrier energies which correlates with  $Q_{\text{bu}}$

# What can we learn from OM3?

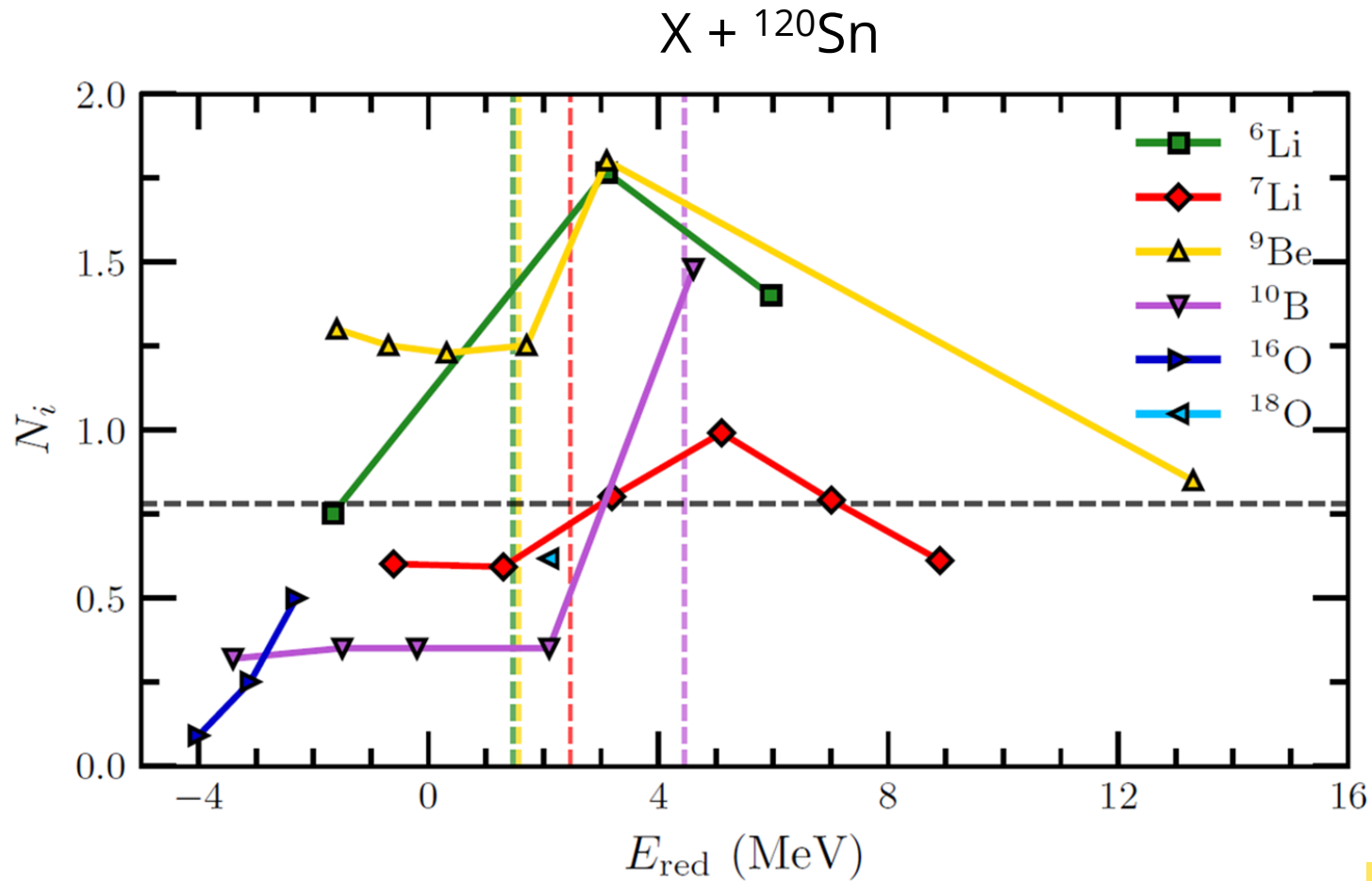


$$E_b^{\text{op}} \geq V_B + |Q_{\text{bu}}|$$

| Nucleus            | Cluster             | $Q_{\text{bu}}$ (MeV) |
|--------------------|---------------------|-----------------------|
| ${}^4\text{He}$    | -                   | -19.813               |
| ${}^{10}\text{Be}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$    | $a + t$             | -2.468                |
| ${}^9\text{Be}$    | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$    | $a + d$             | -1.474                |
| ${}^6\text{He}$    | $a + n + n$         | -0.975                |
| ${}^8\text{B}$     | ${}^7\text{Be} + p$ | -0.136                |

The **imaginary strength** presents a **peak structure** at above-barrier energies which correlates with  $Q_{\text{bu}}$

# What can we learn from OM3?



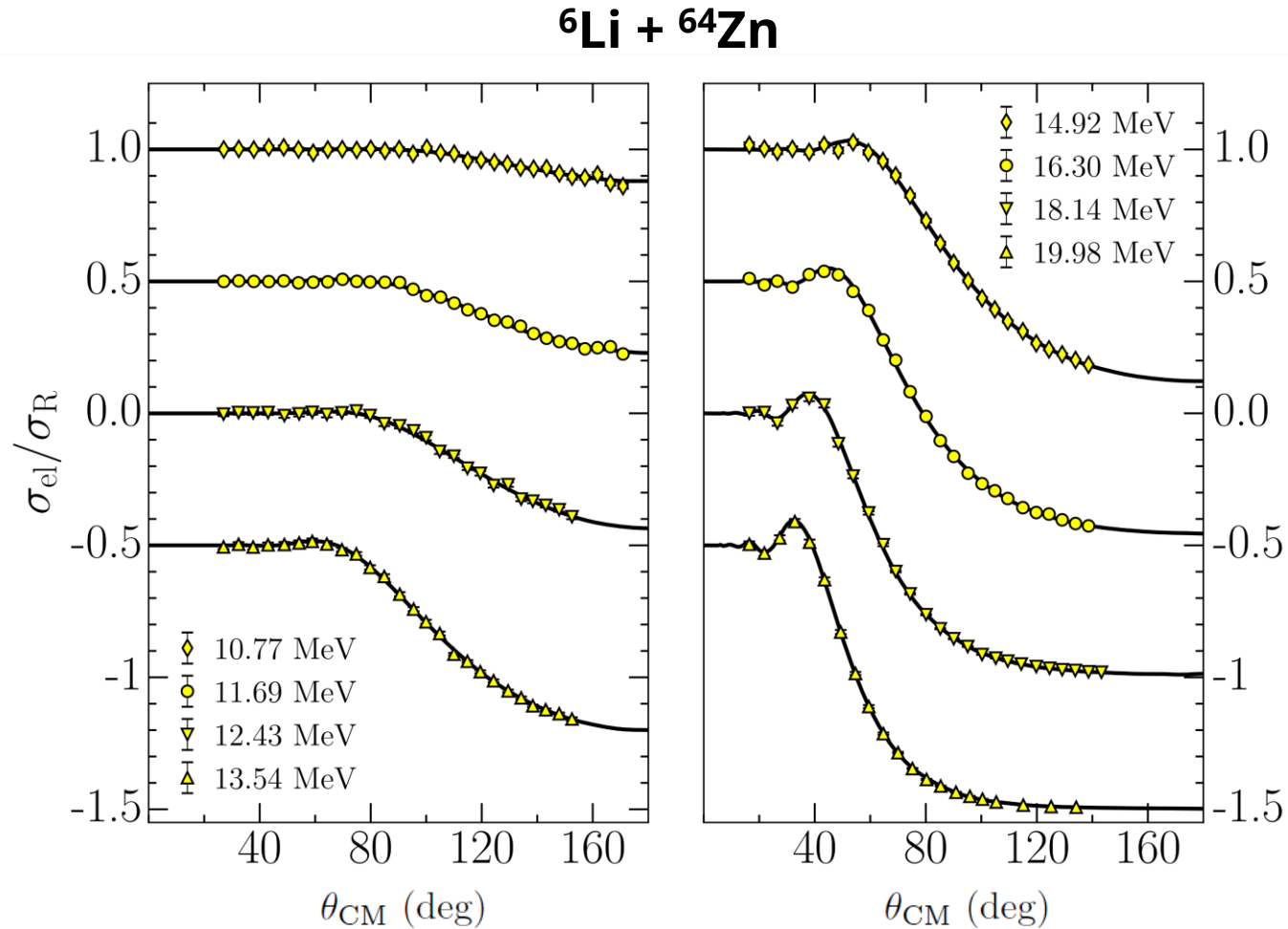
*M. A. G. Alvarez et al. Phys. Rev. C 100, 064602 (2019)*

| Nucleus           | Cluster             | $Q_{\text{bu}}$ (MeV) |
|-------------------|---------------------|-----------------------|
| ${}^{16}\text{O}$ | $a + a + a + a$     | -7.162                |
| ${}^{10}\text{B}$ | ${}^9\text{Be} + n$ | -6.812                |
| ${}^7\text{Li}$   | $a + t$             | -2.468                |
| ${}^9\text{Be}$   | $a + a + n$         | -1.665                |
| ${}^6\text{Li}$   | $a + d$             | -1.474                |

$$E_b^{\text{op}} \geq V_B + |Q_{\text{bu}}|$$

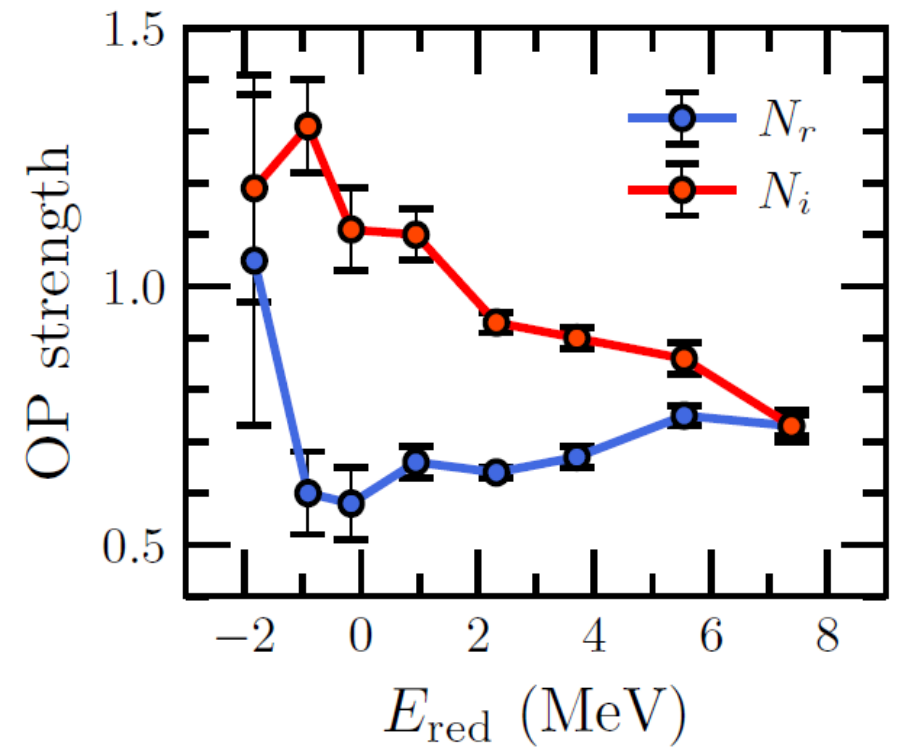
This **systematic result** was previously observed on a heavier target ( ${}^{120}\text{Sn}$ )

# What can we learn from OM4?



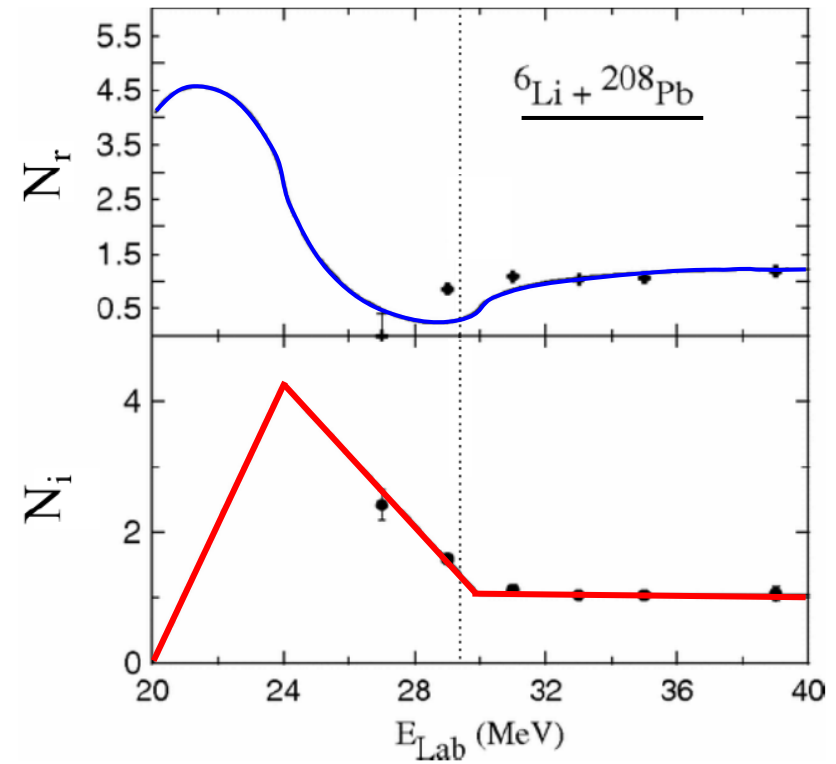
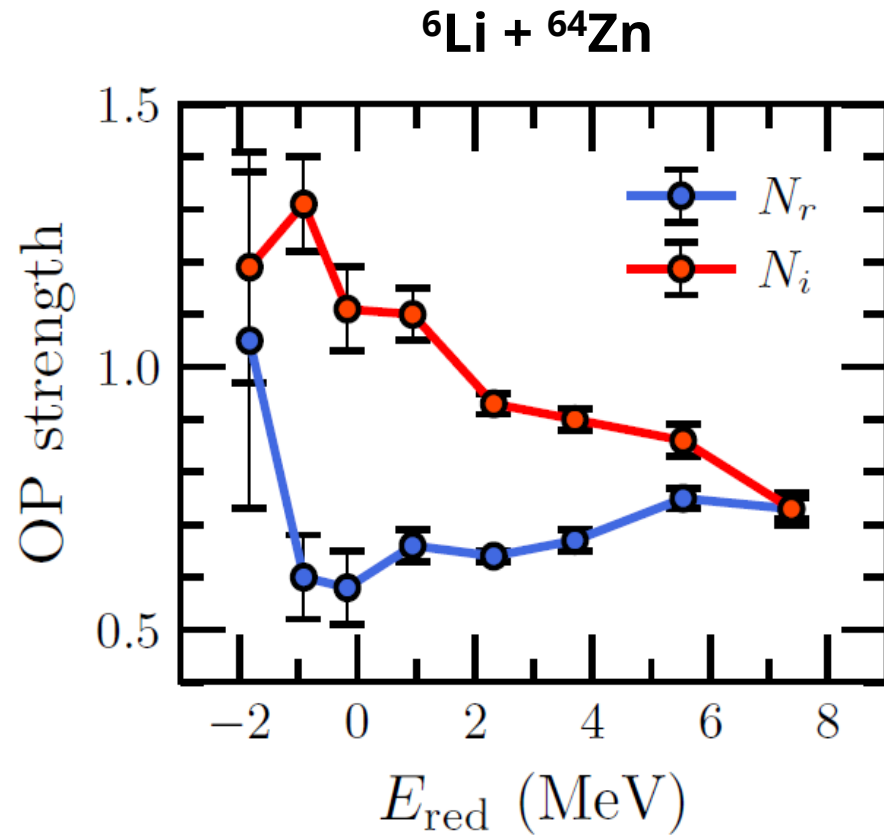
$\chi^2_{\text{red}}$  values from 0.69 to 2.34

$$U_N(R) = N_r V_{\text{SPP2}} + i N_i V_{\text{SPP2}}$$



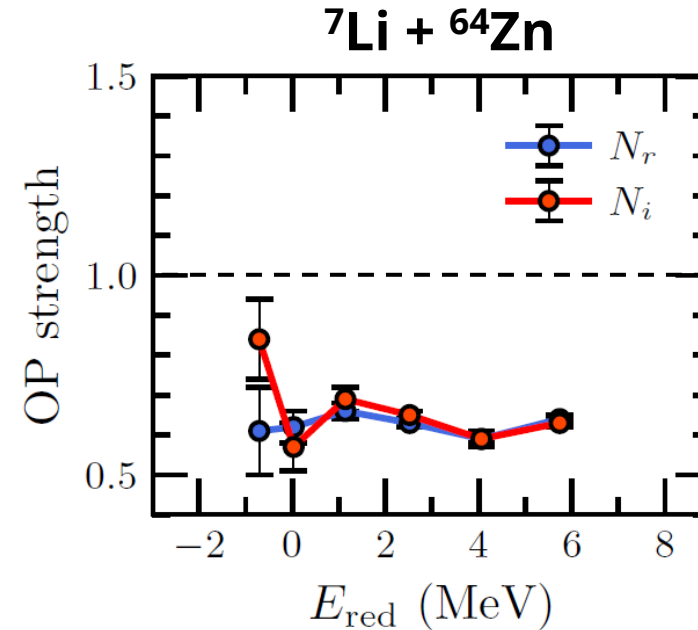
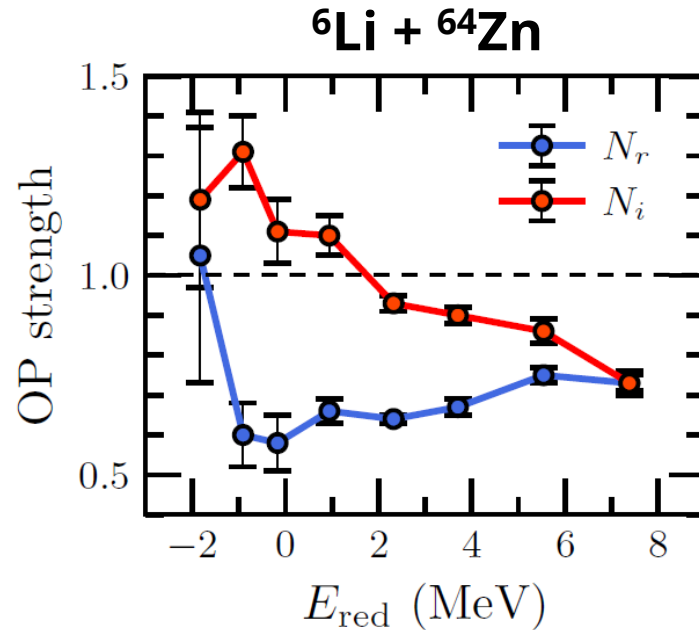
**Dispersion relation**

# What can we learn from OM4?



**Breakup Threshold Anomaly**

# What can we learn from OM4?



TA, BTA, or other ?

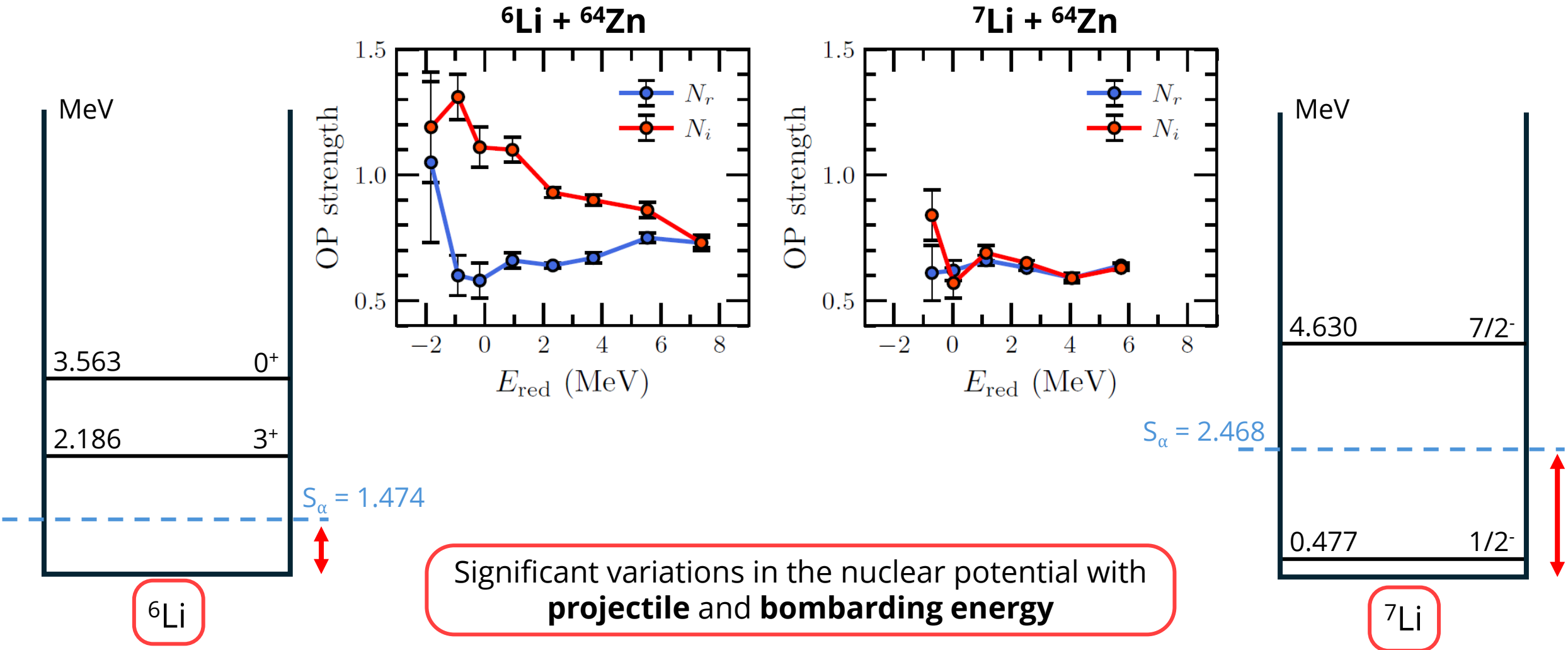


**BTA** is more pronounced for  ${}^6\text{Li}$

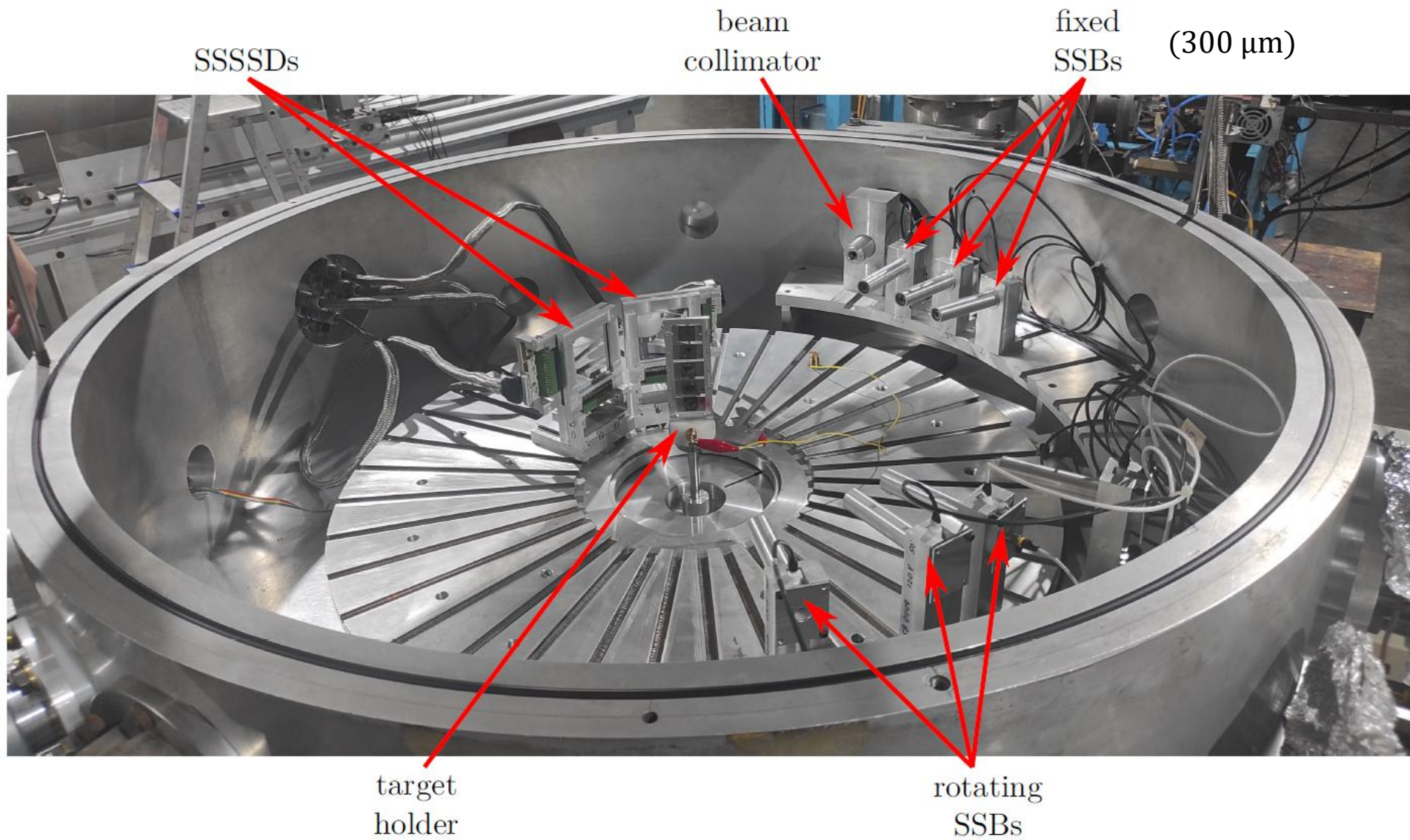
Lower binding energy

No excited states below breakup threshold

# What can we learn from OM4?



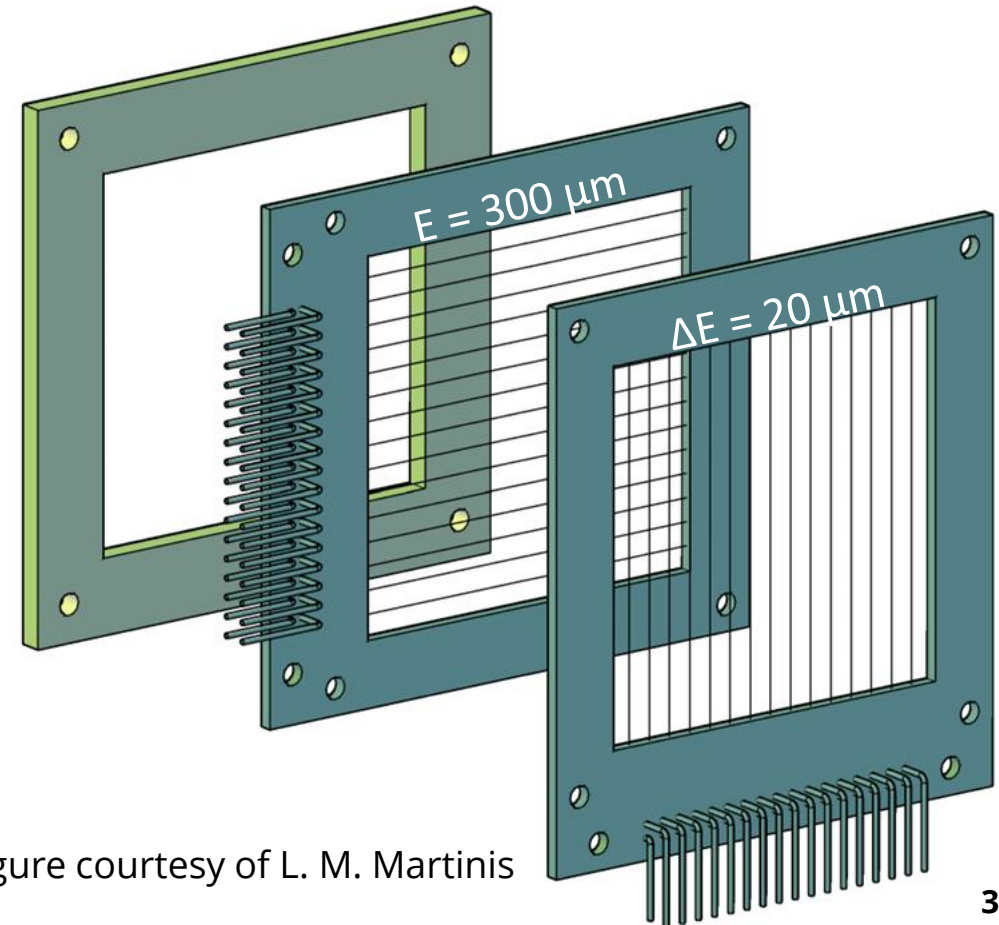
# $^{12,13}\text{C} + ^{64}\text{Zn}$ measurements



## Detection system

Single Sided Silicon Strip Detectors

16 strips  
50 mm x 50 mm



**S**ilicon **T**elescopes **A**rray for **R**eactions  
(STAR)

Figure courtesy of L. M. Martinis

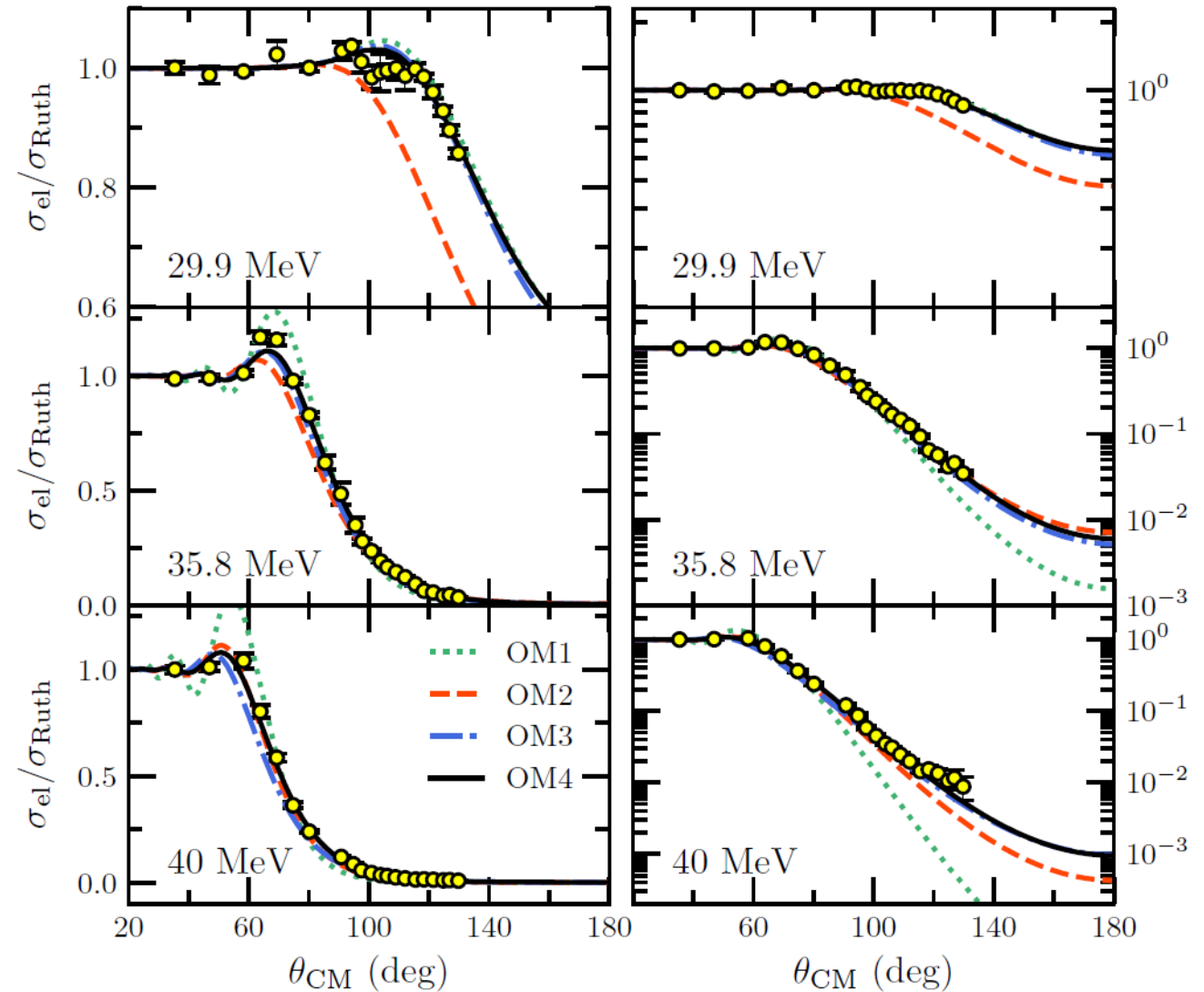
# 1. Elastic scattering: $^{12}\text{C} + ^{64}\text{Zn}$

| 29.9 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|----------|-----------------|-----------------|-----------------------|
| OM1      | 1               | –               | 4.6                   |
| OM2      | 1               | 0.78            | 213                   |
| OM3      | 1               | $0.05 \pm 0.01$ | 2.1                   |
| OM4      | $0.95 \pm 0.08$ | $0.07 \pm 0.03$ | 2.0                   |

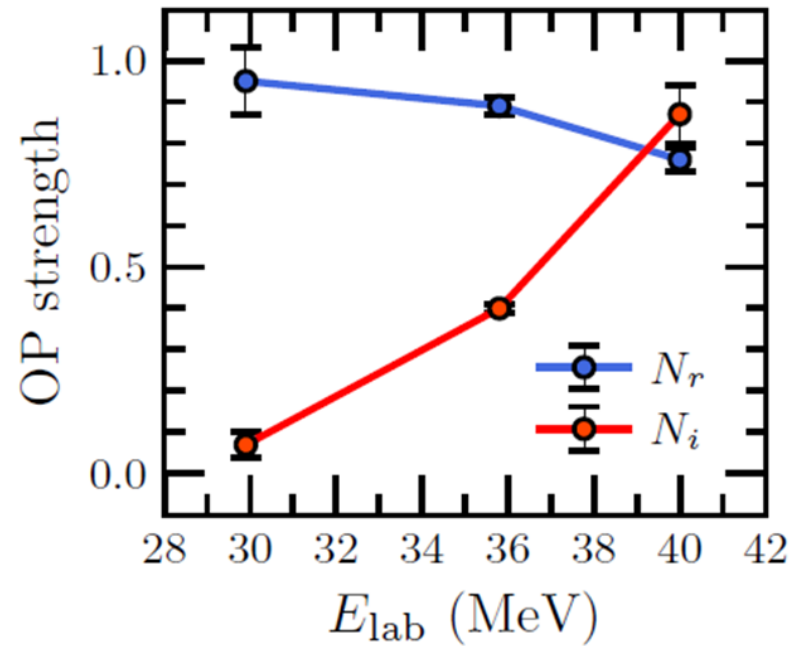
| 35.8 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|----------|-----------------|-----------------|-----------------------|
| OM1      | 1               | –               | 67                    |
| OM2      | 1               | 0.78            | 22                    |
| OM3      | 1               | $0.48 \pm 0.08$ | 15                    |
| OM4      | $0.89 \pm 0.02$ | $0.40 \pm 0.01$ | 2.7                   |

| 40 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|--------|-----------------|-----------------|-----------------------|
| OM1    | 1               | –               | 110                   |
| OM2    | 1               | 0.78            | 26                    |
| OM3    | 1               | $1.48 \pm 0.25$ | 15                    |
| OM4    | $0.76 \pm 0.03$ | $0.87 \pm 0.07$ | 2.4                   |

**Strong rise in the imaginary strength**

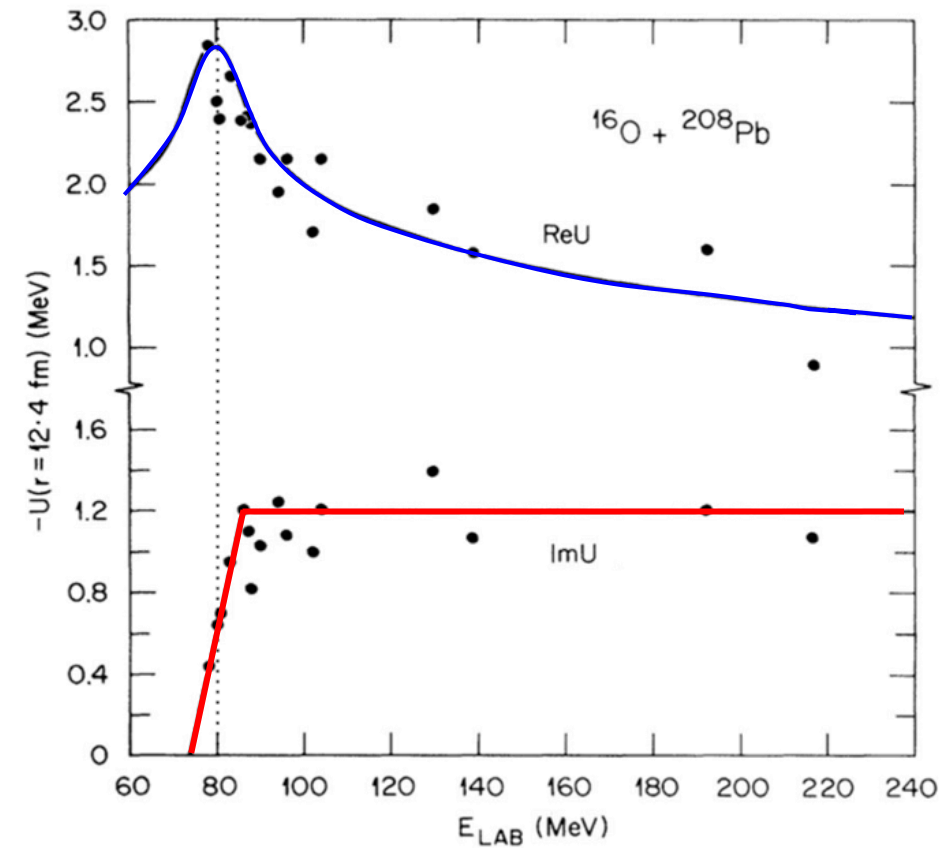


# 1. Elastic scattering: $^{12}\text{C} + ^{64}\text{Zn}$



**Threshold anomaly pattern**

M. A. Nagarajan et al., *Phys. Rev. Lett.* **54**, 1136 (1985)



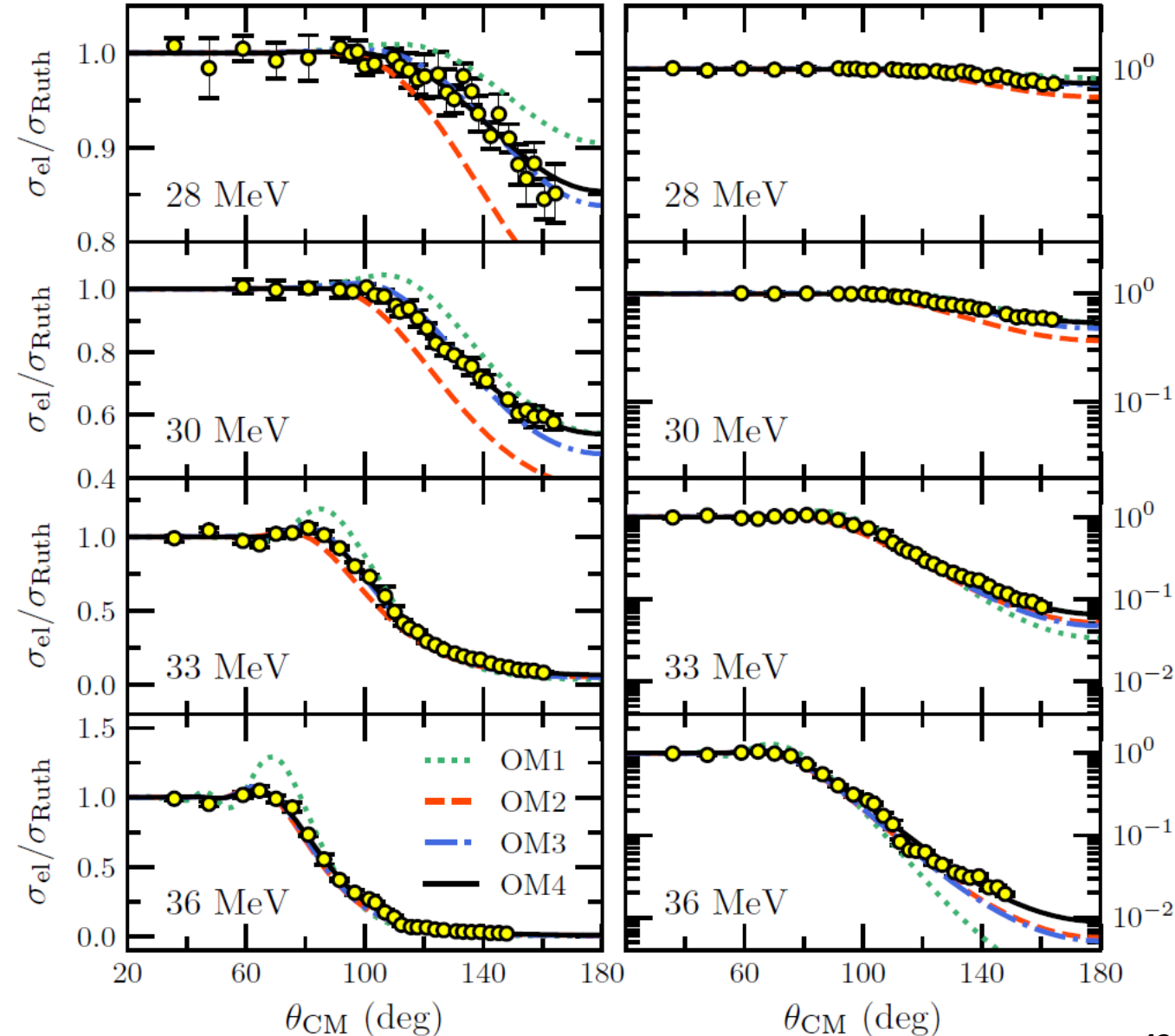
# 1. Elastic scattering: $^{13}\text{C} + ^{64}\text{Zn}$

| 28 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|--------|-----------------|-----------------|-----------------------|
| OM1    | 1               | –               | 3.9                   |
| OM2    | 1               | 0.78            | 10                    |
| OM3    | 1               | $0.25 \pm 0.02$ | 0.7                   |
| OM4    | $0.73 \pm 0.21$ | $0.34 \pm 0.06$ | 0.6                   |

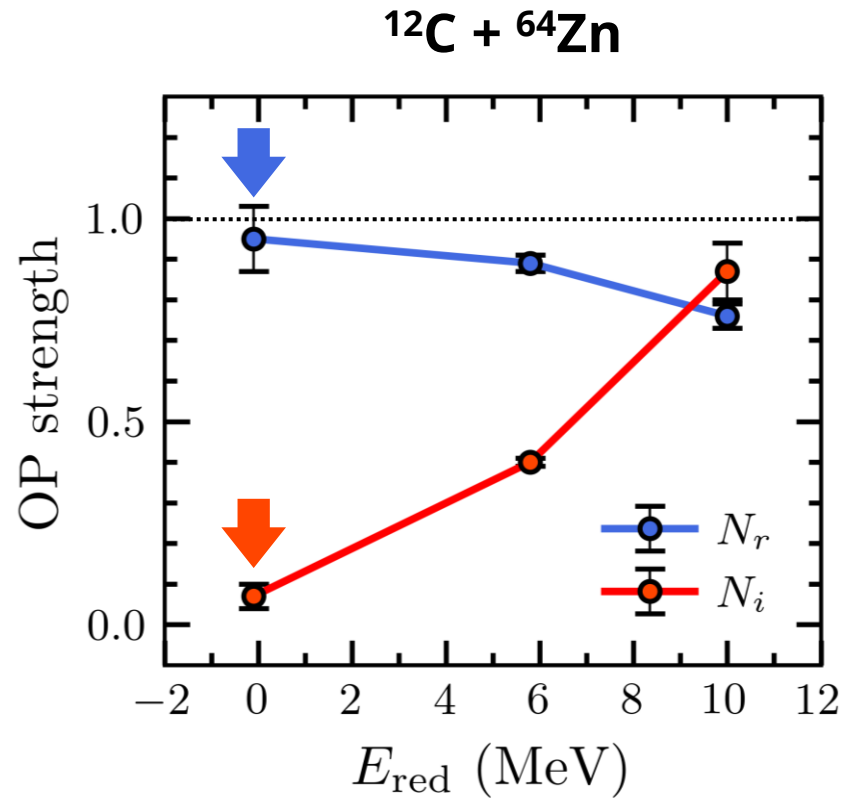
| 30 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|--------|-----------------|-----------------|-----------------------|
| OM1    | 1               | –               | 21                    |
| OM2    | 1               | 0.78            | 55                    |
| OM3    | 1               | $0.21 \pm 0.03$ | 4.0                   |
| OM4    | $0.69 \pm 0.04$ | $0.42 \pm 0.02$ | 0.4                   |

| 33 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|--------|-----------------|-----------------|-----------------------|
| OM1    | 1               | –               | 40                    |
| OM2    | 1               | 0.78            | 24                    |
| OM3    | 1               | $0.37 \pm 0.06$ | 12                    |
| OM4    | $0.86 \pm 0.02$ | $0.46 \pm 0.03$ | 2.0                   |

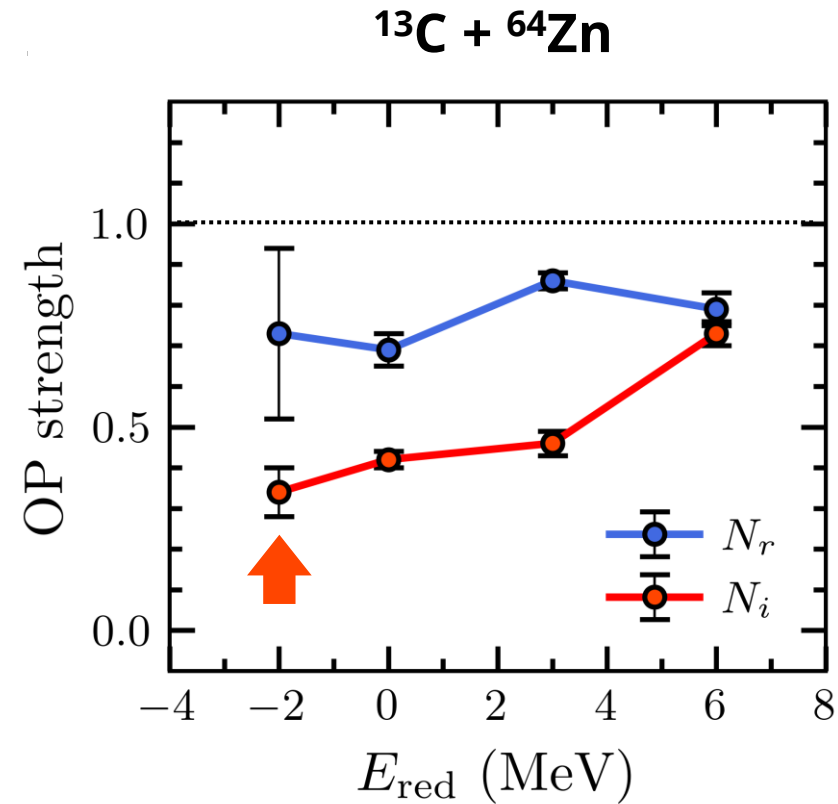
| 36 MeV | $N_r$           | $N_i$           | $\chi^2_{\text{red}}$ |
|--------|-----------------|-----------------|-----------------------|
| OM1    | 1               | –               | 79                    |
| OM2    | 1               | 0.78            | 23                    |
| OM3    | 1               | $0.68 \pm 0.12$ | 22                    |
| OM4    | $0.79 \pm 0.04$ | $0.73 \pm 0.03$ | 5.9                   |



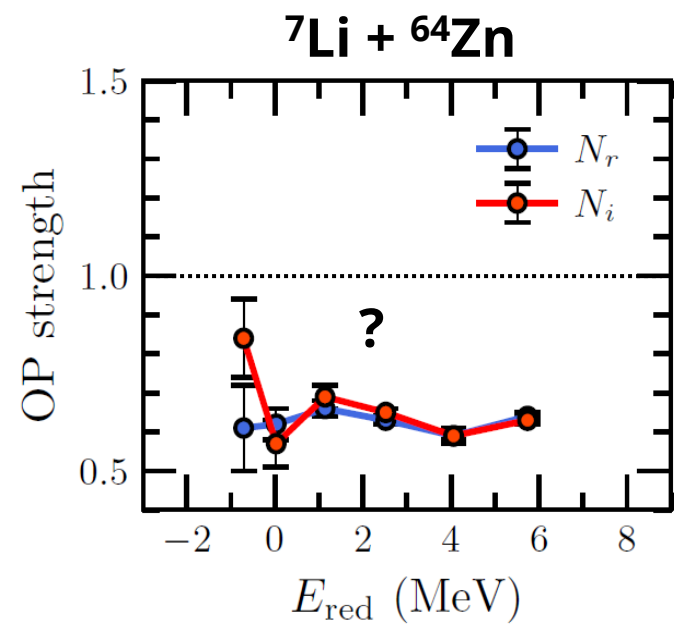
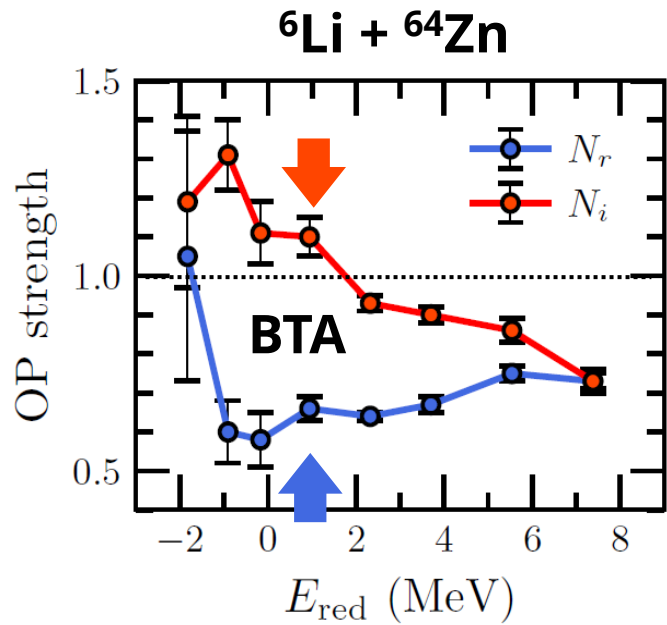
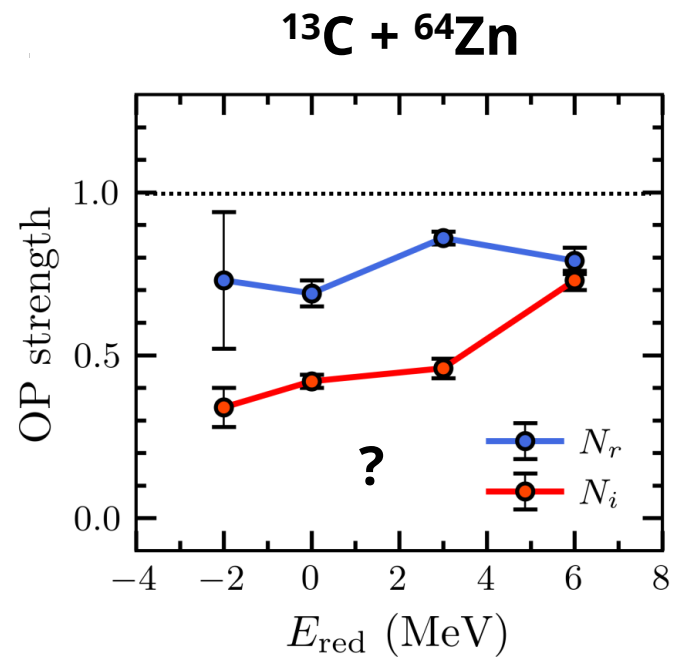
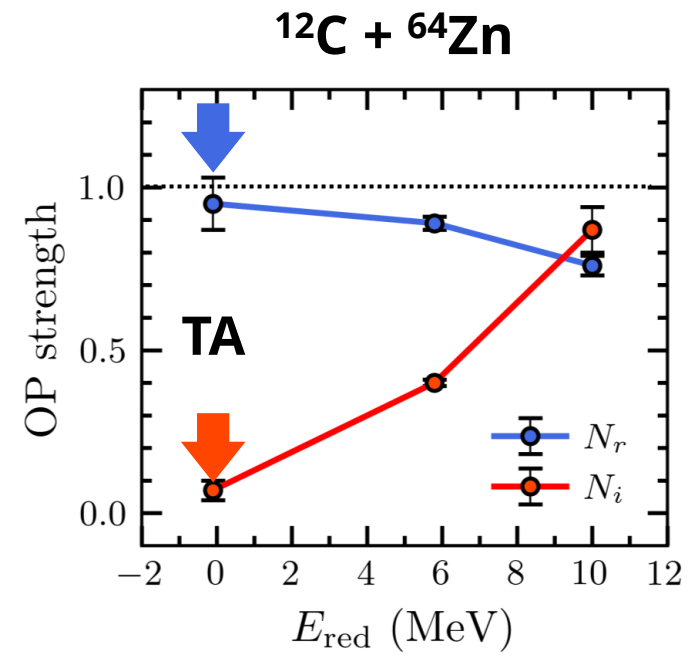
# $^{12,13}\text{C} + ^{64}\text{Zn}$ comparison



Threshold anomaly pattern



Significant imaginary strength  
at sub-barrier energies



**Further investigation on  
the dispersion relation**

OP strengths ( $N_r$ ,  $N_i$ )



Opening/closing reaction channels



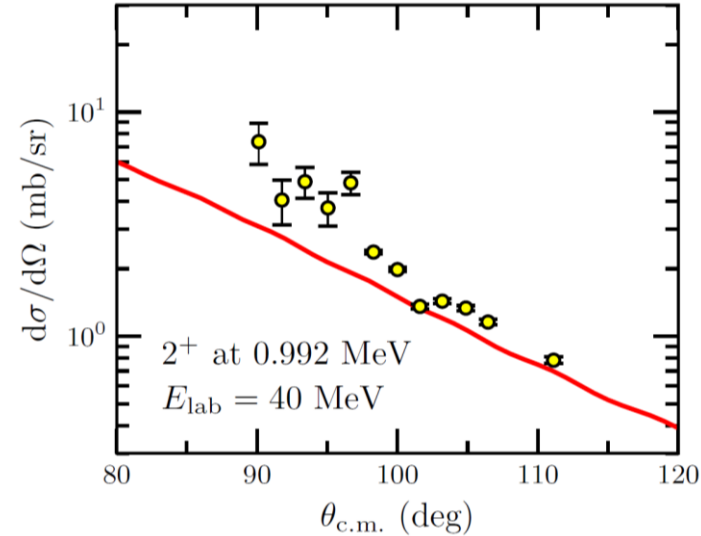
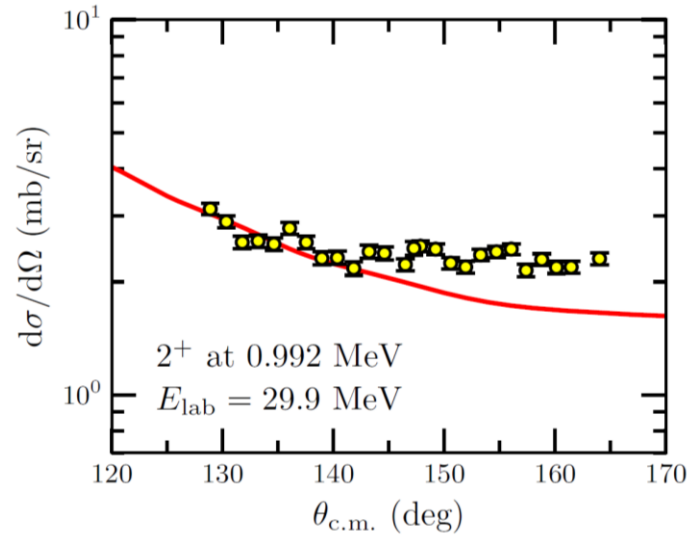
Nuclear structure

# THANK YOU

lgarrido2@us.es

## 2. Inelastic scattering: $2^+$ state at 0.992 MeV ( $^{64}\text{Zn}$ )

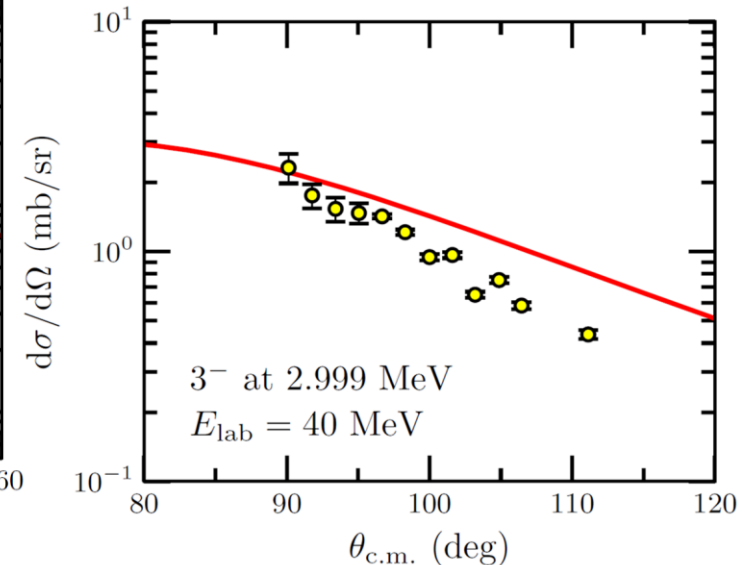
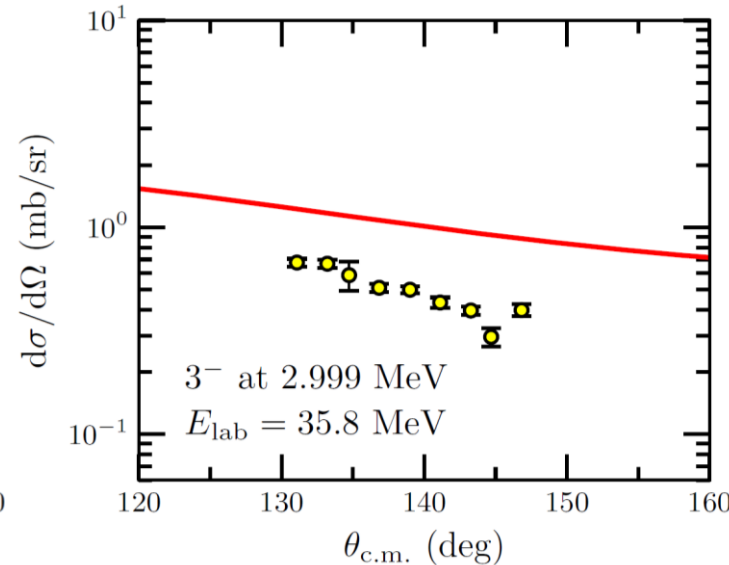
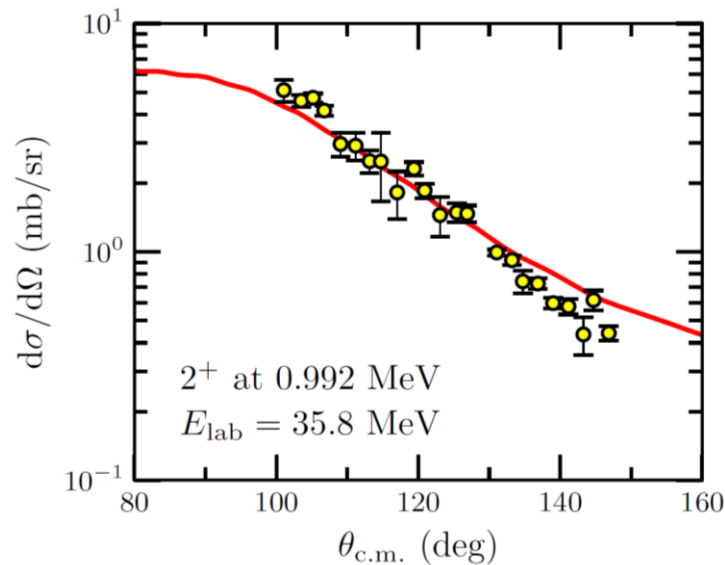
## 3. Inelastic scattering: $3^-$ state at 2.999 MeV ( $^{64}\text{Zn}$ )



| nucleus          | $J^\pi$ | $E^*$ | $\lambda$ | $B(E\lambda)$ | $\delta_\lambda$ |
|------------------|---------|-------|-----------|---------------|------------------|
| $^{64}\text{Zn}$ | $2^+$   | 0.992 | 2         | 168           | 1.18             |
| $^{64}\text{Zn}$ | $3^-$   | 2.999 | 3         | 34            | 1.02             |
| $^{12}\text{C}$  | $2^+$   | 4.440 | 2         | 3.97          | 1.42             |
| $^{12}\text{C}$  | $3^-$   | 9.641 | 3         | 0.251         | 1.66             |

(MeV)      ( $10^{-3} \text{ e}^2\text{b}^\lambda$ )    (fm)

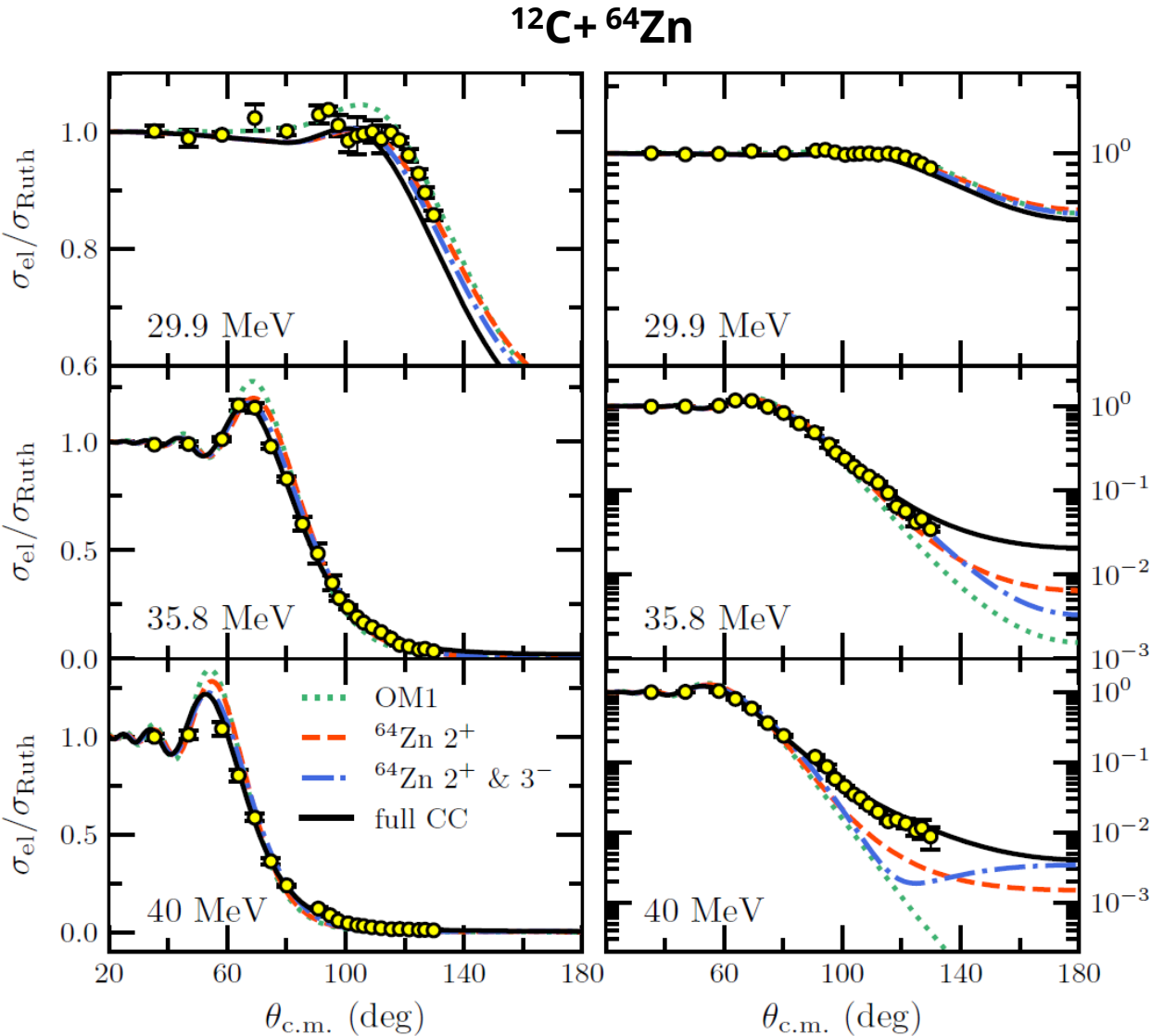
The **OM1** internal potential was used as the **initial interaction** and the reaction channels were coupled



2. Inelastic scattering: 2<sup>+</sup> state at 0.992 MeV (<sup>64</sup>Zn)

3. Inelastic scattering: 3<sup>-</sup> state at 2.999 MeV (<sup>64</sup>Zn)

L. Garrido-Gómez et al., *Phys. Rev. C* 113, 044607 (2026)



| nucleus          | $J^\pi$        | $E^*$ | $\lambda$ | $B(E\lambda)$ | $\delta_\lambda$ |
|------------------|----------------|-------|-----------|---------------|------------------|
| <sup>64</sup> Zn | 2 <sup>+</sup> | 0.992 | 2         | 168           | 1.18             |
| <sup>64</sup> Zn | 3 <sup>-</sup> | 2.999 | 3         | 34            | 1.02             |
| <sup>12</sup> C  | 2 <sup>+</sup> | 4.440 | 2         | 3.97          | 1.42             |
| <sup>12</sup> C  | 3 <sup>-</sup> | 9.641 | 3         | 0.251         | 1.66             |

(MeV)

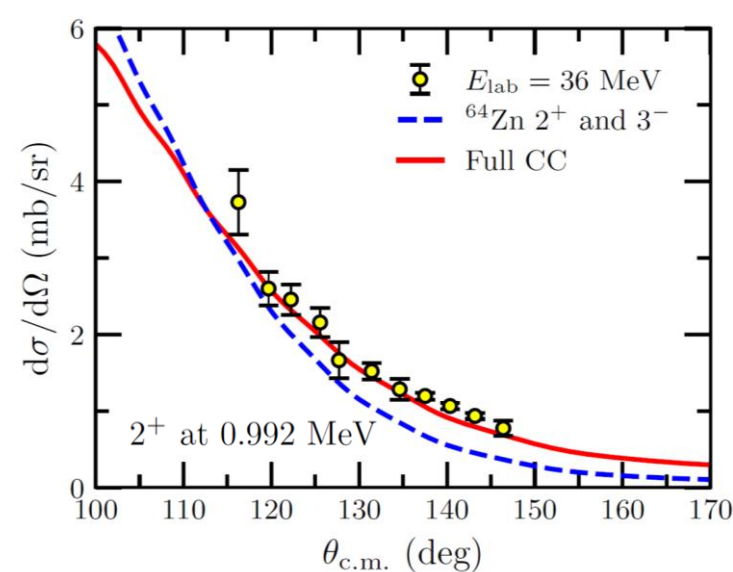
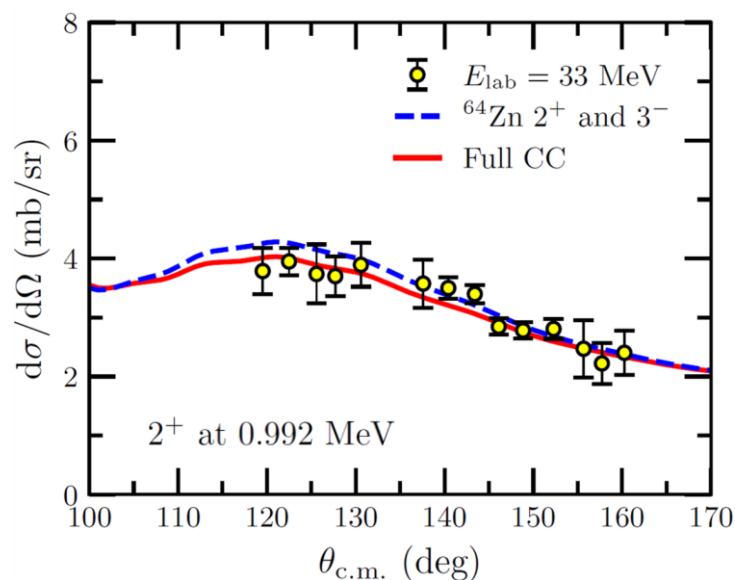
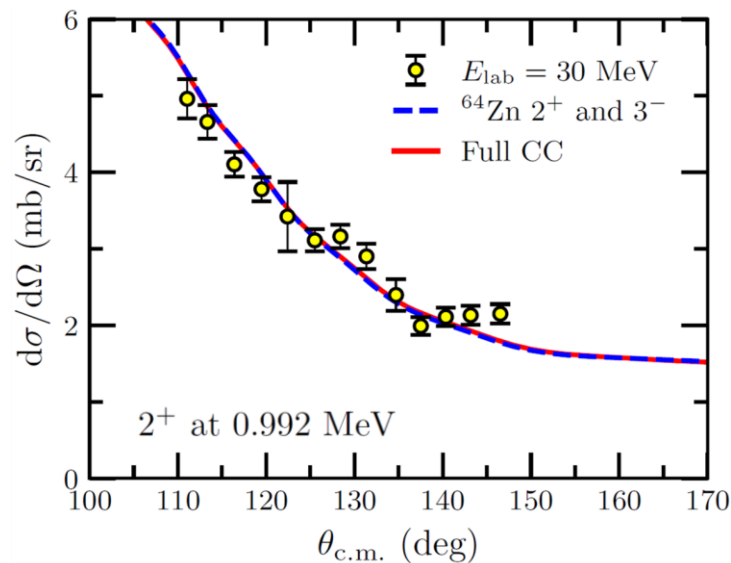
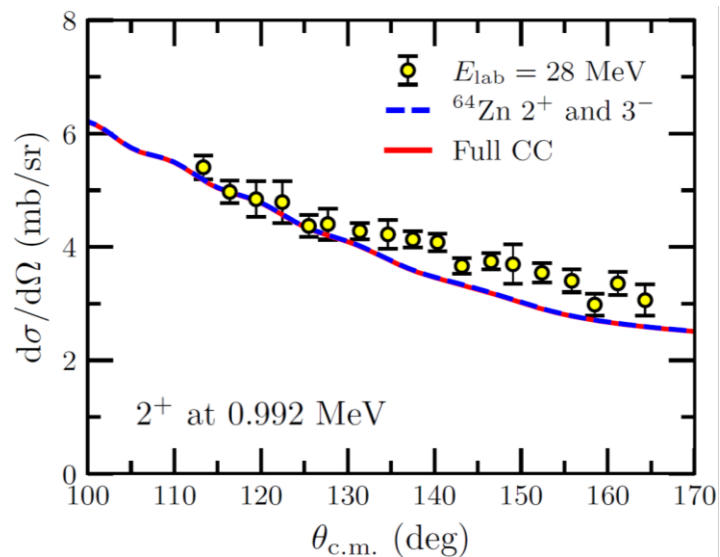
(10<sup>-3</sup> e<sup>2</sup>b<sup>λ</sup>)

(fm)

The **OM1** internal potential was used as the **initial interaction** and the reaction channels were coupled

## 2. Inelastic scattering $^{13}\text{C} + ^{64}\text{Zn}$ : $2^+$ state at 0.992 MeV ( $^{64}\text{Zn}$ )

## 3. Inelastic scattering $^{13}\text{C} + ^{64}\text{Zn}$ : $3^-$ state at 2.999 MeV ( $^{64}\text{Zn}$ )



| nucleus          | $J^\pi$ | $E^*$ | $\lambda$ | $B(E\lambda)$ | $\delta_\lambda$ |
|------------------|---------|-------|-----------|---------------|------------------|
| $^{64}\text{Zn}$ | $2^+$   | 0.992 | 2         | 168           | 1.18             |
| $^{64}\text{Zn}$ | $3^-$   | 2.999 | 3         | 34            | 1.02             |
| $^{13}\text{C}$  | $1/2^+$ | 3.089 | 1         | 0.152         | 0.116            |
| $^{13}\text{C}$  | $3/2^-$ | 3.685 | 2         | 1.27          | 1.12             |

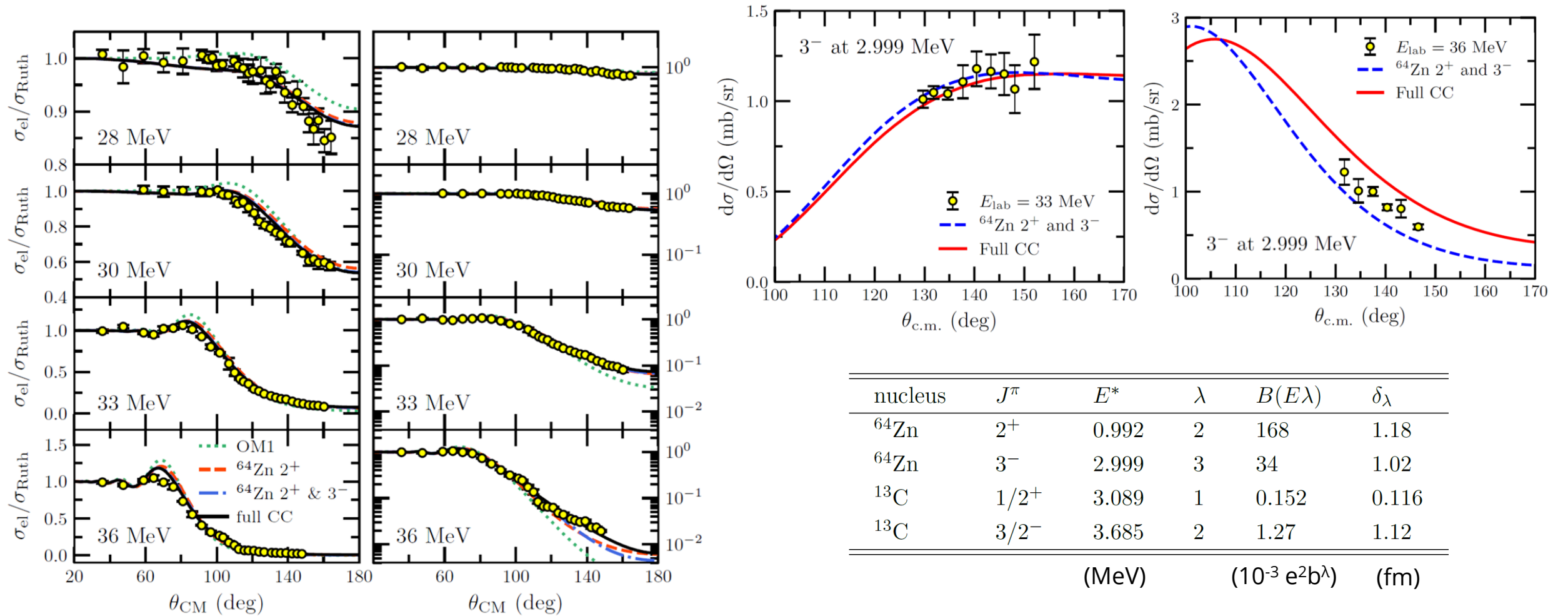
(MeV)                      ( $10^{-3} \text{ e}^2\text{b}^\lambda$ )                      (fm)

The **OM1** internal potential was used as the **initial interaction** and the reaction channels were coupled

**Good description** of the inelastic angular distributions  
**(no free parameters)**

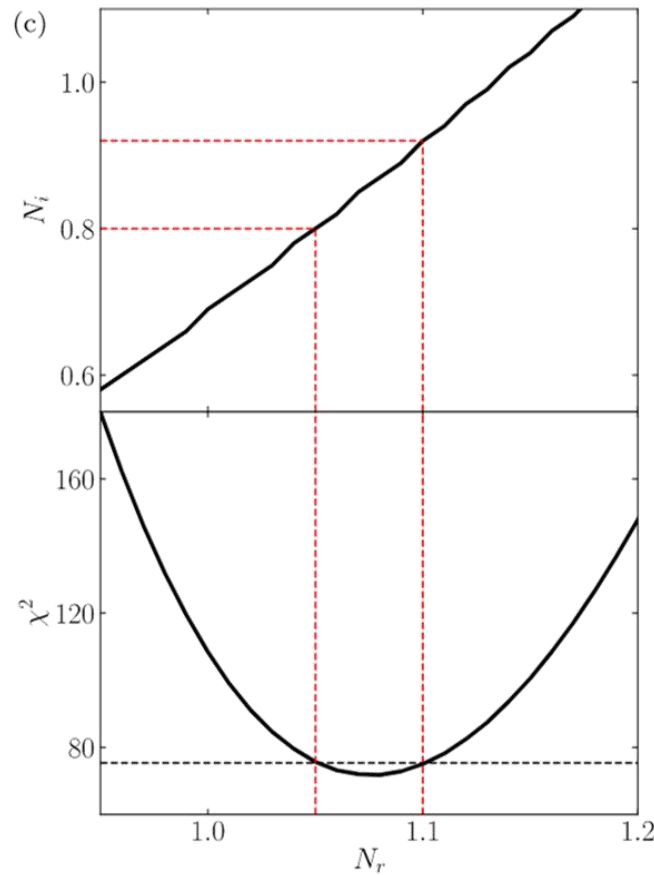
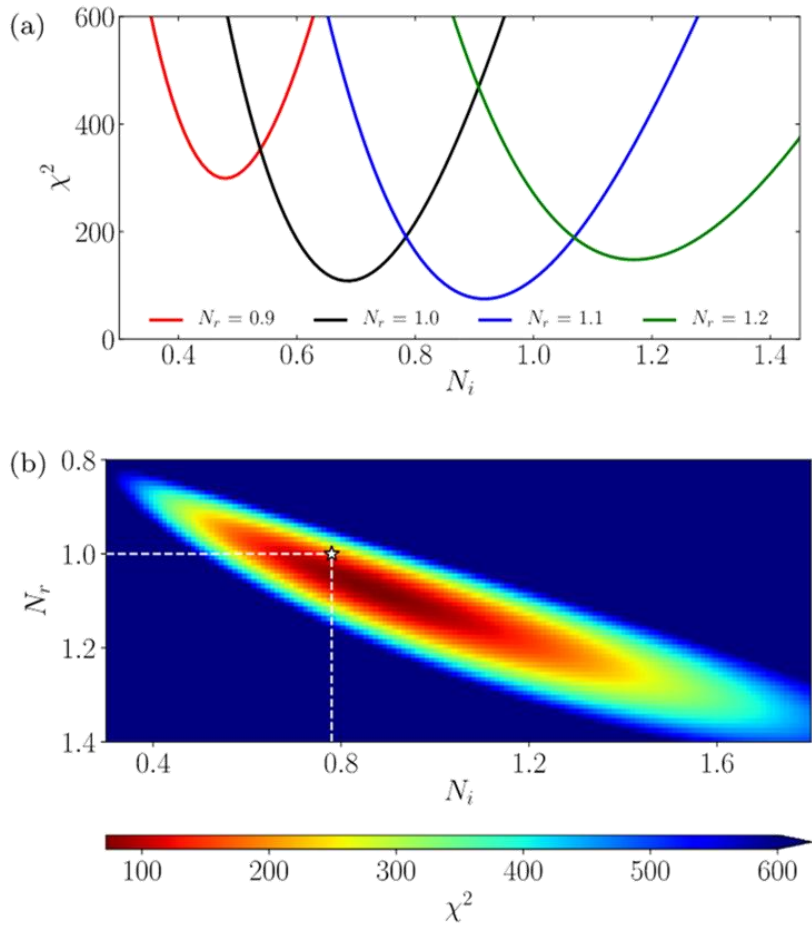
## 2. Inelastic scattering $^{13}\text{C} + ^{64}\text{Zn}$ : $2^+$ state at 0.992 MeV ( $^{64}\text{Zn}$ )

### 3. Inelastic scattering $^{13}\text{C} + ^{64}\text{Zn}$ : $3^-$ state at 2.999 MeV ( $^{64}\text{Zn}$ )



**Good description of the inelastic angular distributions  
(no free parameters)**

# Uncertainties in OP strengths



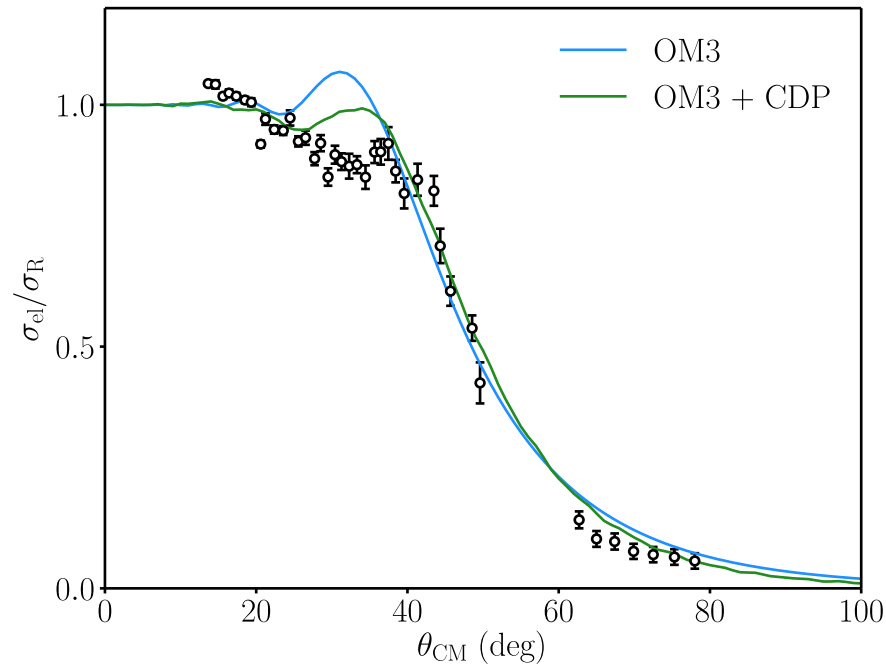
W. H. Press et al.  
*Numerical Recipes: The Art of Scientific Computing*, 3rd ed.  
 Cambridge University Press, 2007.

| $\Delta\chi^2$ as a Function of Confidence Level $p$ and Number of Parameters of Interest $\nu$ |       |      |      |      |      |      |
|-------------------------------------------------------------------------------------------------|-------|------|------|------|------|------|
| $p$                                                                                             | $\nu$ |      |      |      |      |      |
|                                                                                                 | 1     | 2    | 3    | 4    | 5    | 6    |
| 68.27%                                                                                          | 1.00  | 2.30 | 3.53 | 4.72 | 5.89 | 7.04 |
| 90%                                                                                             | 2.71  | 4.61 | 6.25 | 7.78 | 9.24 | 10.6 |
| 95.45%                                                                                          | 4.00  | 6.18 | 8.02 | 9.72 | 11.3 | 12.8 |
| 99%                                                                                             | 6.63  | 9.21 | 11.3 | 13.3 | 15.1 | 16.8 |
| 99.73%                                                                                          | 9.00  | 11.8 | 14.2 | 16.3 | 18.2 | 20.1 |
| 99.99%                                                                                          | 15.1  | 18.4 | 21.1 | 23.5 | 25.7 | 27.9 |

$$\chi^2_{\min} + 2.3 * \chi^2_{\min} / \nu$$

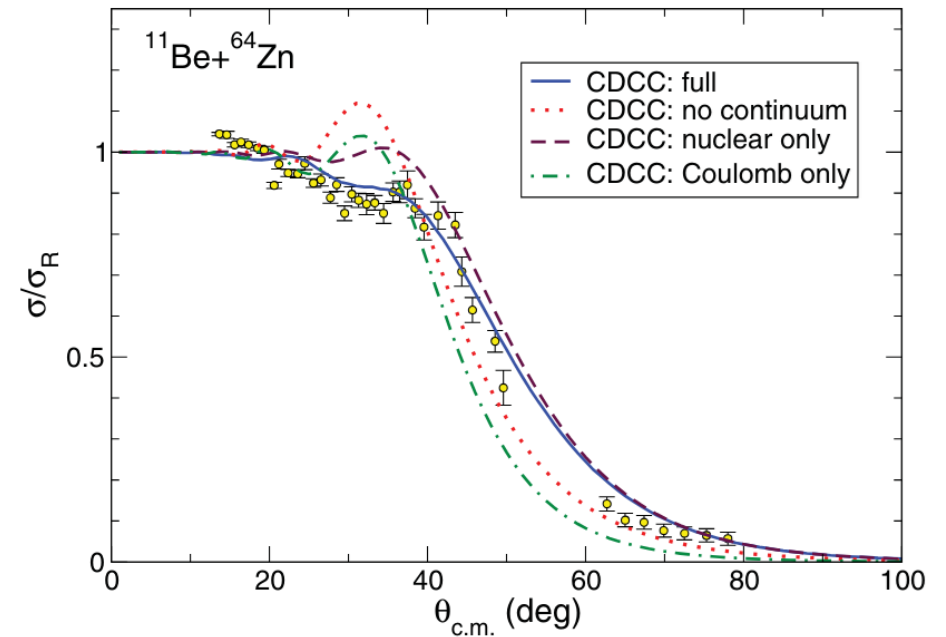
# The case of $^{11}\text{Be} + ^{64}\text{Zn}$

Adding Coulomb effects

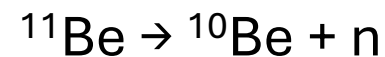
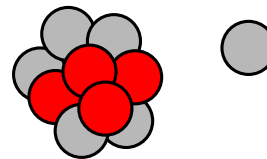


$(\chi^2_{\text{red}} = 30)$   
 $(\chi^2_{\text{red}} = 14)$

Adding complex nuclear effects



$$U_{CDP} \propto \frac{Z_t}{v} \int_{\varepsilon_b}^{\infty} d\varepsilon \frac{dB(E1)}{d\varepsilon}$$



Phys. Rev. C **85**, 054607 (2012)

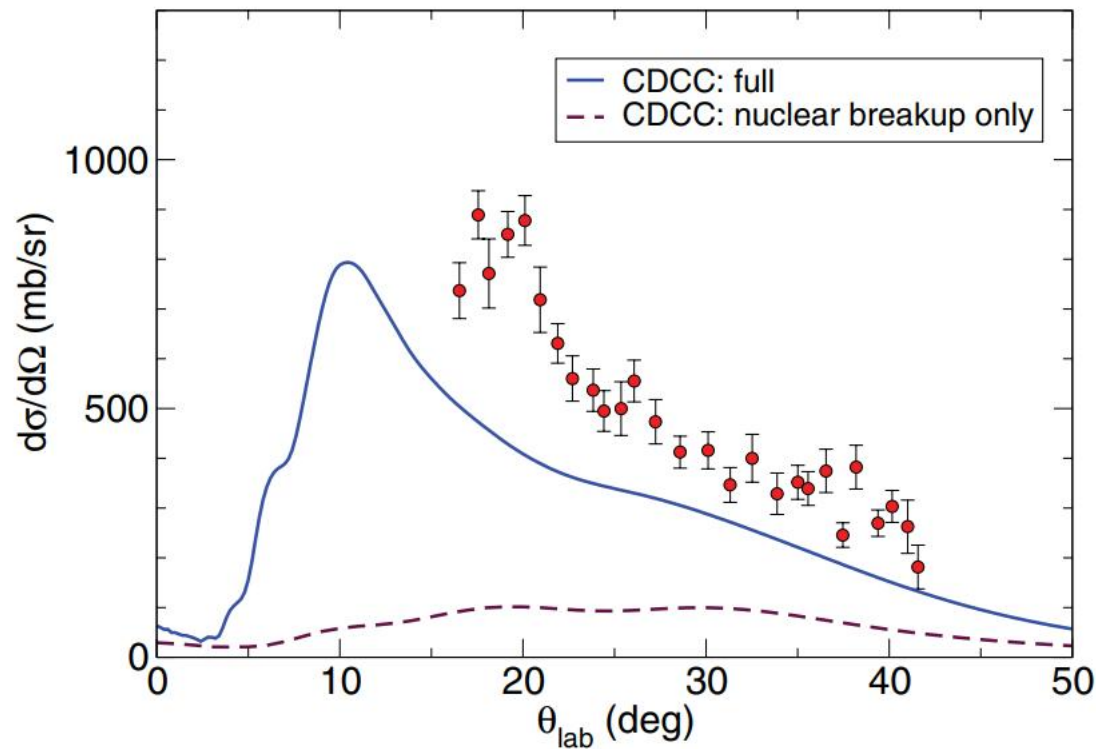
A. Di Pietro *et al.*

# The case of exotic nuclei

Phys. Rev. C **92**, 014604 (2015)

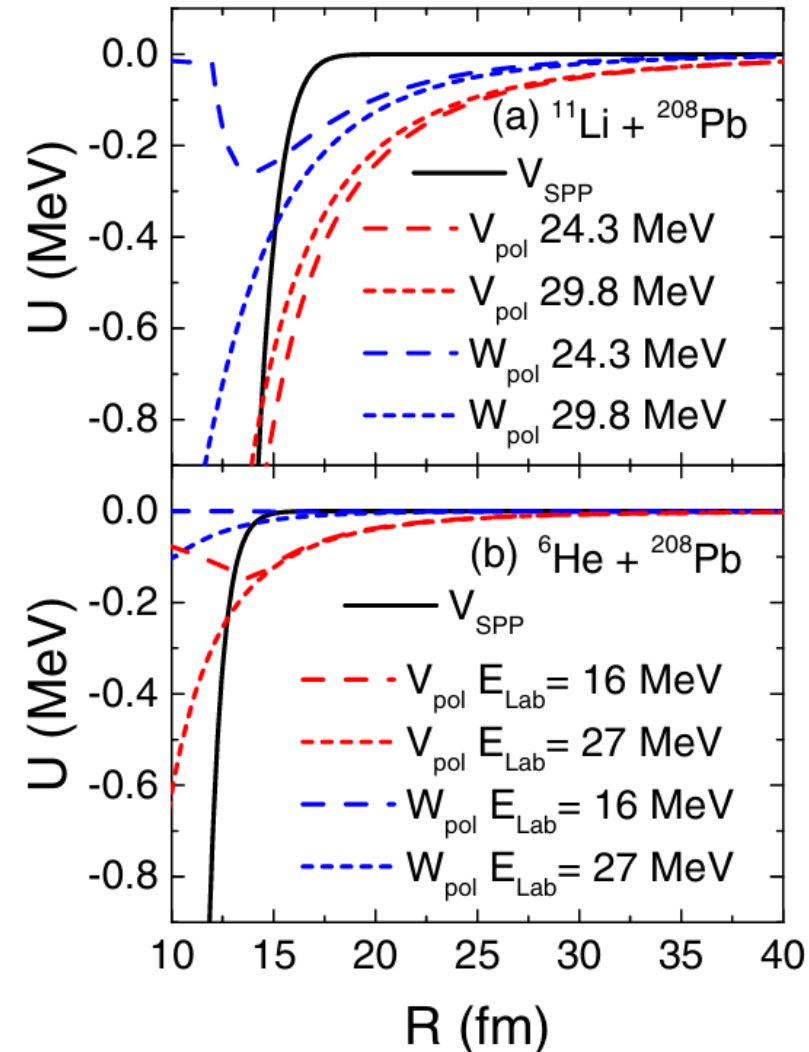
J. P. Fernández-García *et al.*

Breakup CS in  $^{11}\text{Be} + ^{64}\text{Zn}$



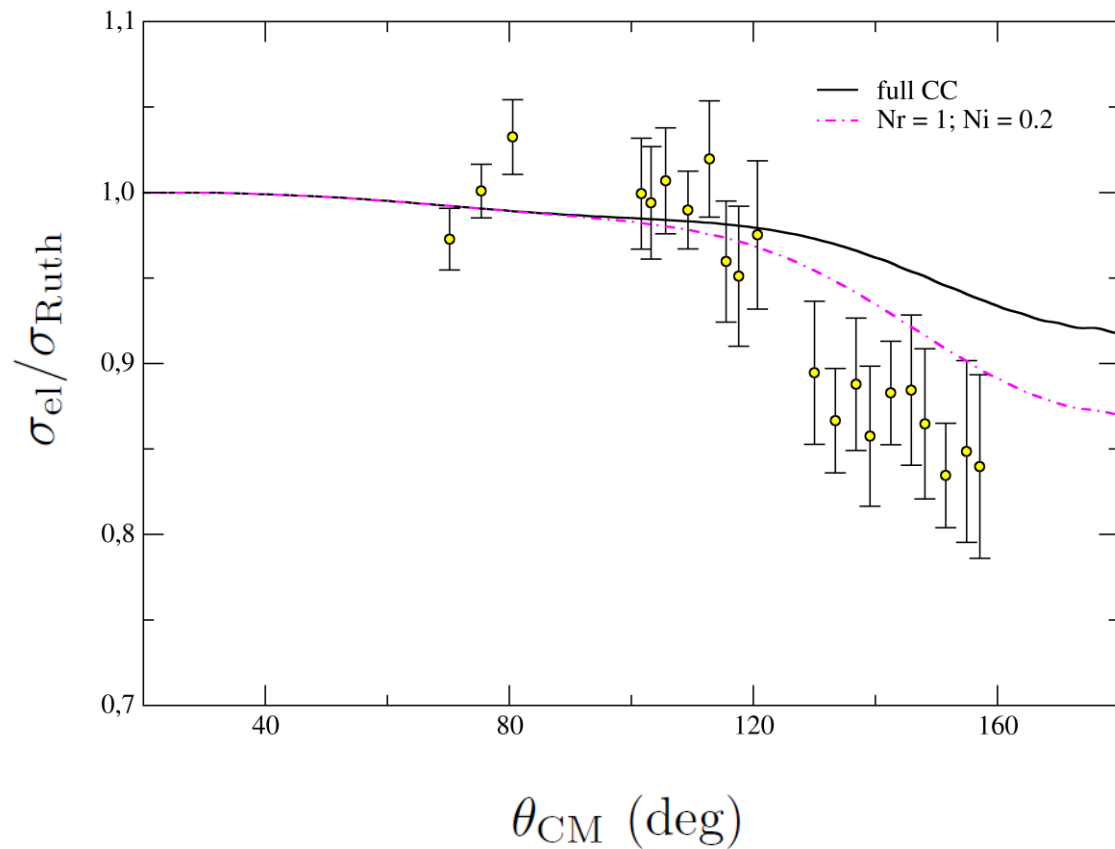
Phys. Rev. C **85**, 054607 (2012)

A. Di Pietro *et al.*

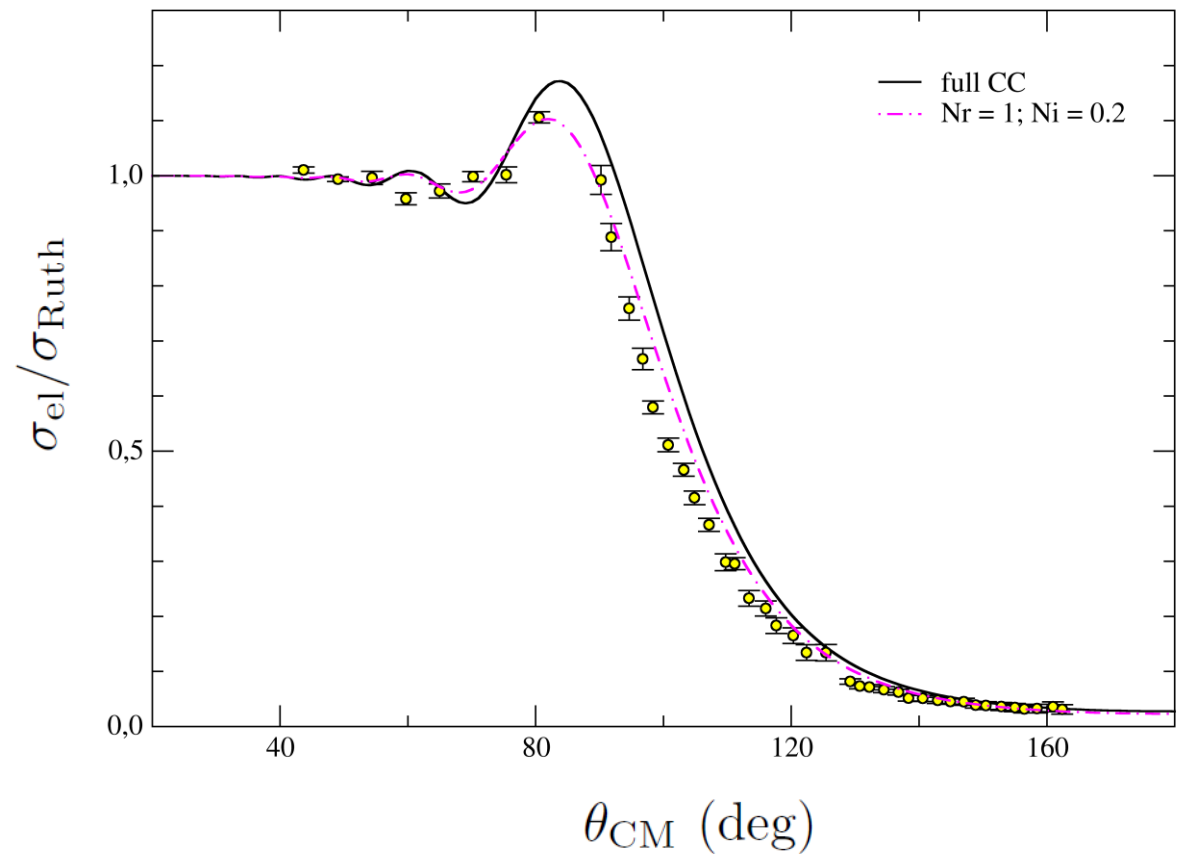


# Adding imaginary strength to CC calculations

$^{12}\text{C}+^{120}\text{Sn}$  @ 38.7 MeV



$^{12}\text{C}+^{120}\text{Sn}$  @ 46.7 MeV



# Adding imaginary strength to CC calculations

$^{13}\text{C} + ^{64}\text{Zn}$  @ 36 MeV

