

Study of neutron-rich β -delayed emitters relevant for understanding the formation of the r -process rare earth-peak around mass $A \sim 160$

Max Pallàs Solís

TU Darmstadt / GSI
Former UPC-INTE

Stellar nucleosynthesis

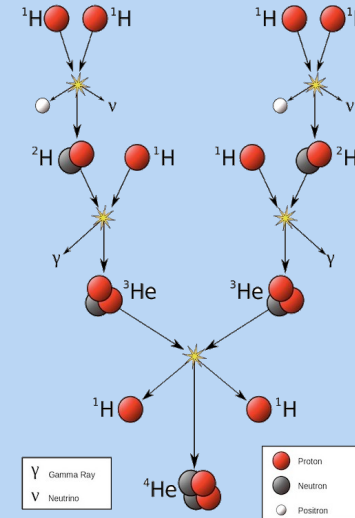
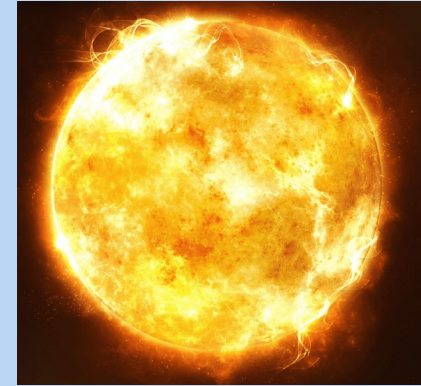
In the core of stars, **nuclear fusion** takes place.

1. H burning (PP-Chain / CNO-cycle)
2. He burning (Triple- α)
3.

+Z

Stops at Fe (Z=26)!

1 IA H Hydrogen 1.008 1																	18 VIIIA He Helium 4.0026 2	
3 Li Lithium 6.94 3	4 Be Beryllium 9.013 4																	10 Ne Neon 20.18 10
11 Na Sodium 22.98 11	12 Mg Magnesium 24.32 12																	18 Ar Argon 39.95 18
19 K Potassium 39.10 19	20 Ca Calcium 40.08 20	21 Sc Scandium 44.96 21	22 Ti Titanium 47.87 22	23 V Vanadium 50.94 23	24 Cr Chromium 51.996 24	25 Mn Manganese 54.94 25	26 Fe Iron 55.84 26	27 Co Cobalt 58.93 27	28 Ni Nickel 58.69 28	29 Cu Copper 63.55 29	30 Zn Zinc 65.38 30	31 Ga Gallium 69.72 31	32 Ge Germanium 72.63 32	33 As Arsenic 74.92 33	34 Se Selenium 78.96 34	35 Br Bromine 79.90 35	36 Kr Krypton 83.80 36	
37 Rb Rubidium 85.47 37	38 Sr Strontium 87.62 38	39 Y Yttrium 88.91 39	40 Zr Zirconium 91.22 40	41 Nb Niobium 92.91 41	42 Mo Molybdenum 95.94 42	43 Tc Technetium [98] 43	44 Ru Ruthenium 101.07 44	45 Rh Rhodium 101.07 45	46 Pd Palladium 106.42 46	47 Ag Silver 107.87 47	48 Cd Cadmium 112.41 48	49 In Indium 114.82 49	50 Sn Tin 118.71 50	51 Sb Antimony 121.76 51	52 Te Tellurium 127.60 52	53 I Iodine 126.91 53	54 Xe Xenon 131.29 54	



Science Learning Hub (Adapted)

And heavy nuclei?

In 1957, the B²FH paper was published, proposing a solution for the formation of heavy elements: The neutron capture reaction (n,γ).

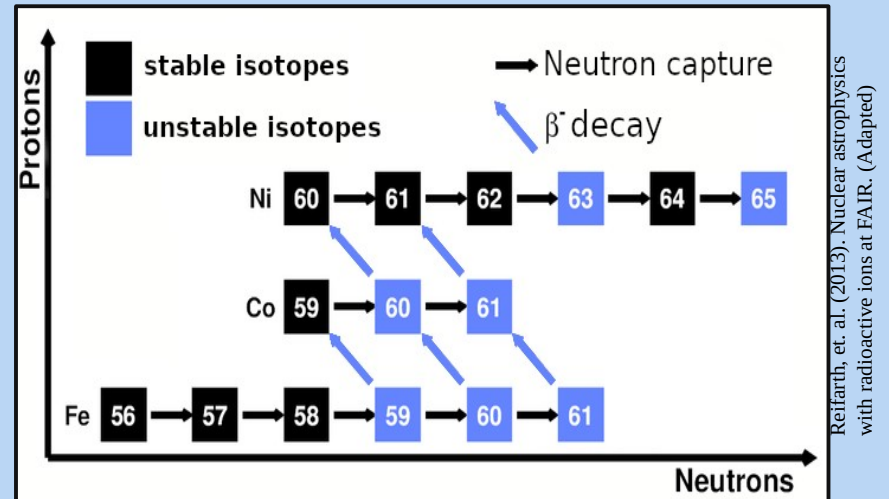
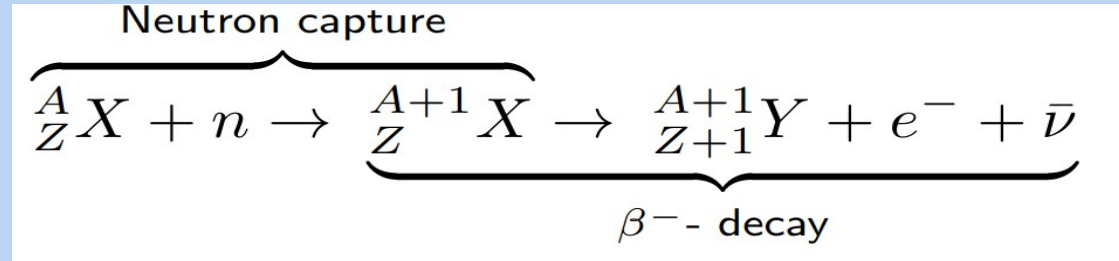
Neutron capture (n,γ)

Tends to instability

β⁻-decay

Tends to stability

Z increase



Neutron capture

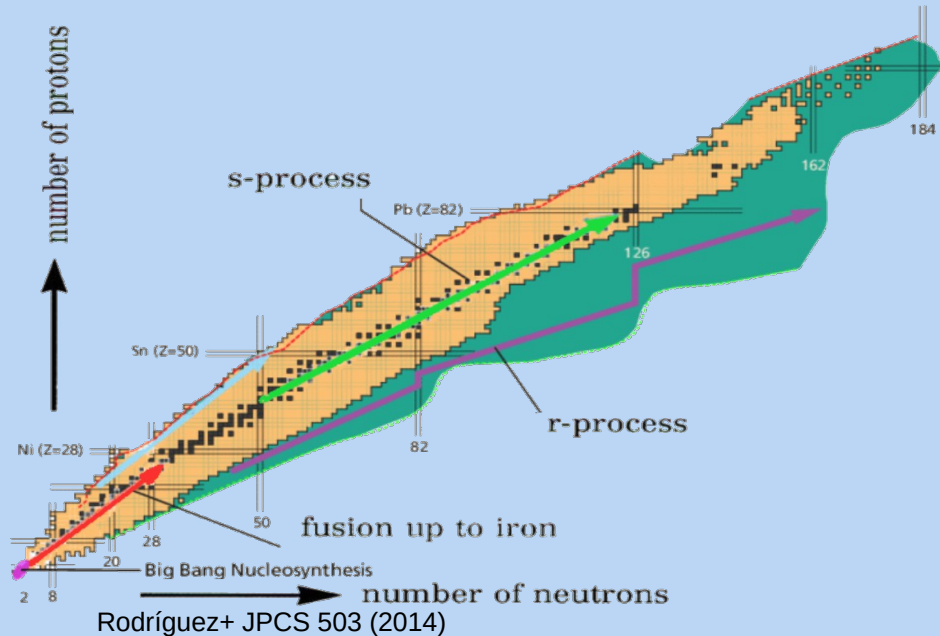
Two processes can be distinguished based on the **relative timescales of neutron capture (T_{neu}) and β -decay (T_{β})**.

Slow neutron-capture process:

- Low neutron density: $T_{\text{Neu}} > T_{\beta}$
- Around the stability line.
- Timescale: stellar burning.

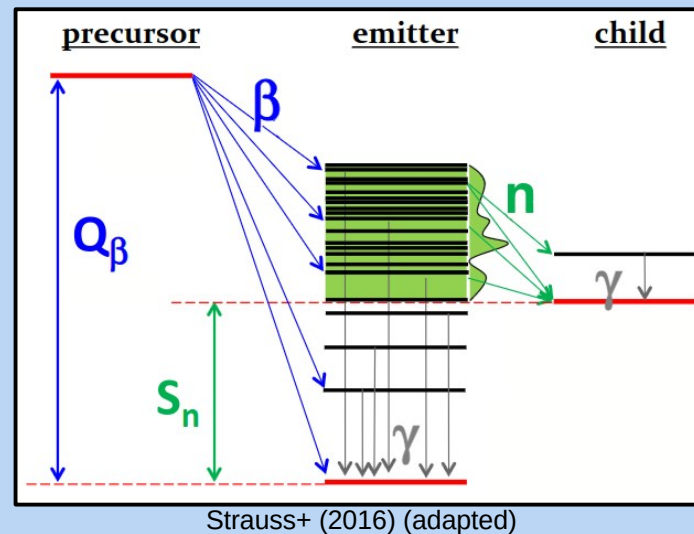
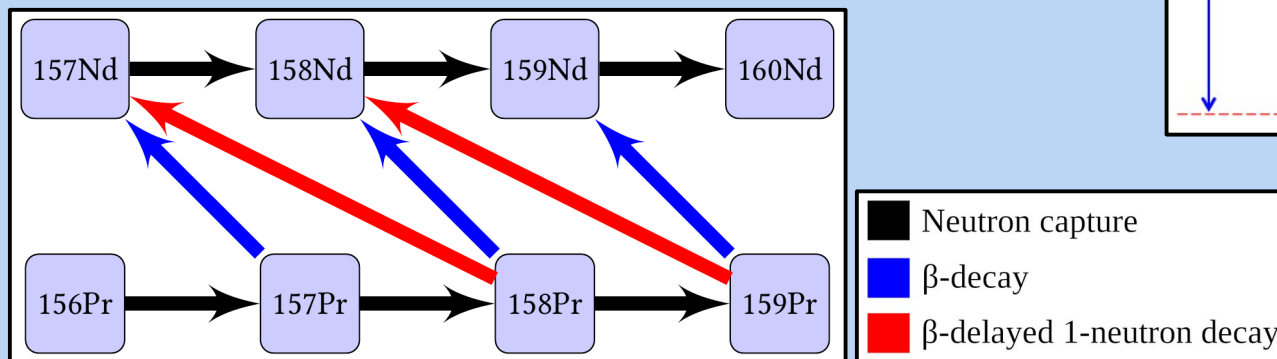
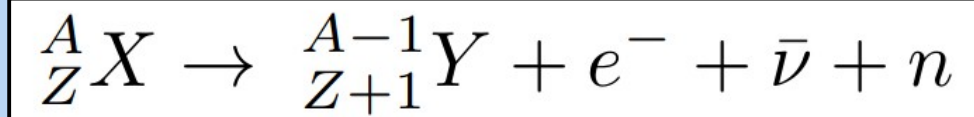
Rapid neutron-capture process:

- High neutron density: $T_{\text{Neu}} \ll T_{\beta}$
- Far from stability line.
- Timescale: μs to ms .



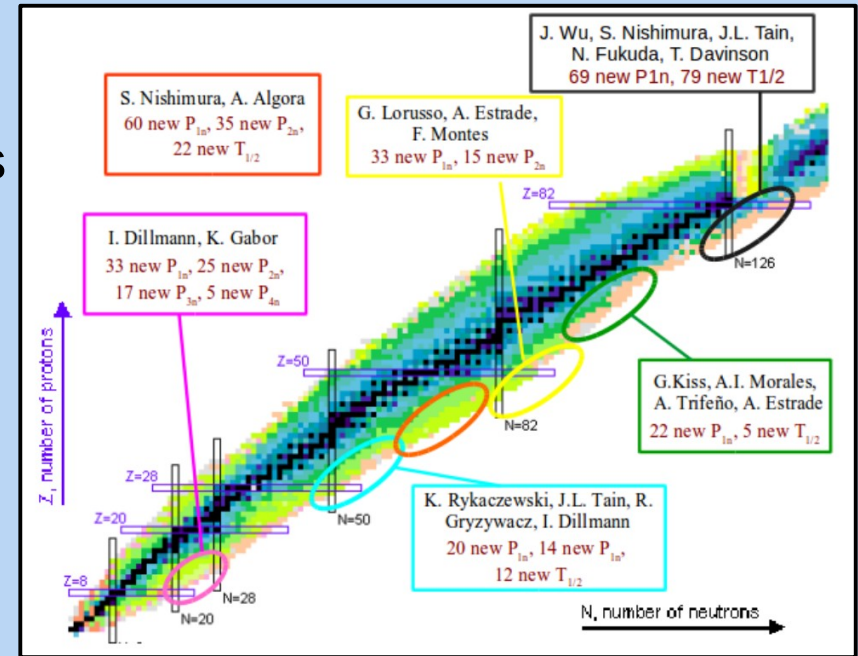
β -delayed neutron emission

- The **β -decay** is characteristic of neutron-rich nuclei.
- The energy available is called Q_β .
- If $Q_\beta > S_{xn} \rightarrow$ **β -delayed neutron emission**.



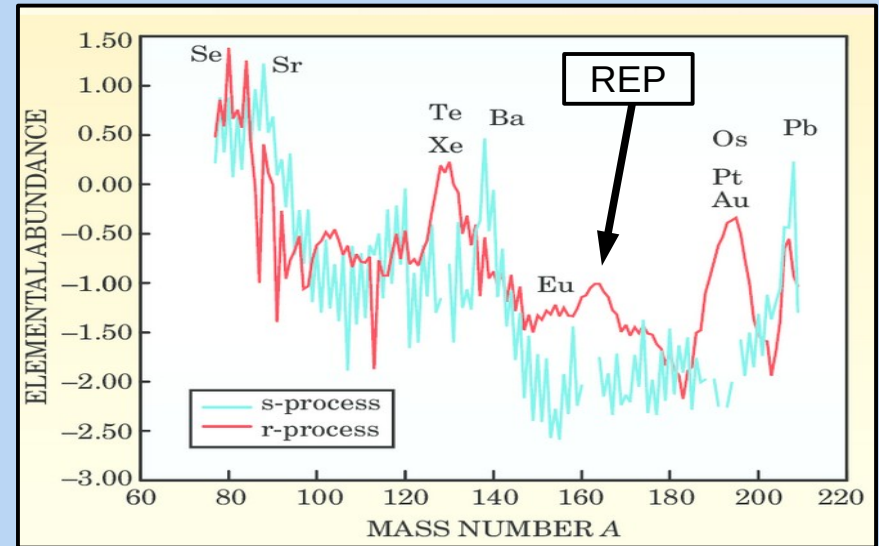
BRIKEN project

- Built the **largest β -delayed neutron detector** in the world to conduct experiments at the RIKEN Nishina Center (Japan).
- Study the **most exotic** experimentally accessible neutron-rich nuclei.
 - **Half-life ($T_{1/2}$)**
 - **Beta-delayed neutron emission probabilities (P_{xn})**
- **International collaboration:** Over 50 participants from 18 international institutions.

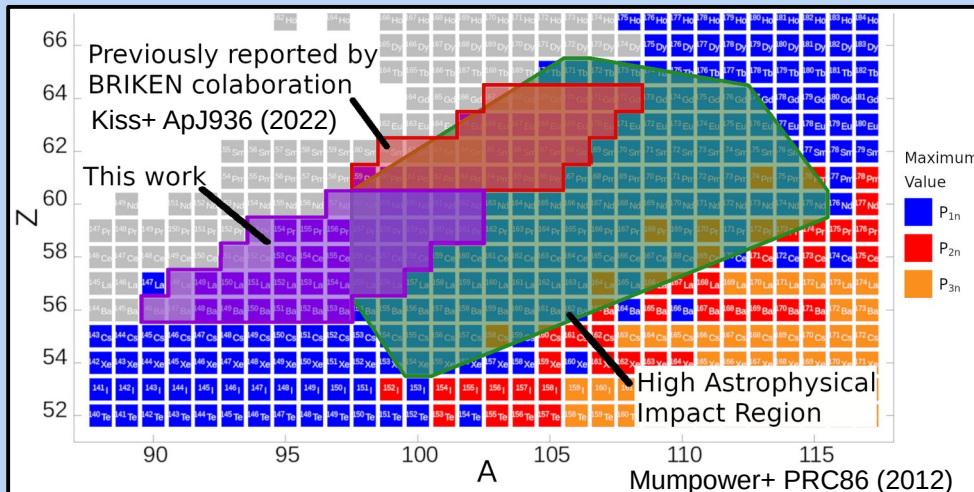


REP relevance

- The **only** r-process peak formed after the **freeze-out**.
- Unique probe** for the study of the **late-time conditions** on the r-process site (e. g. Temperature)



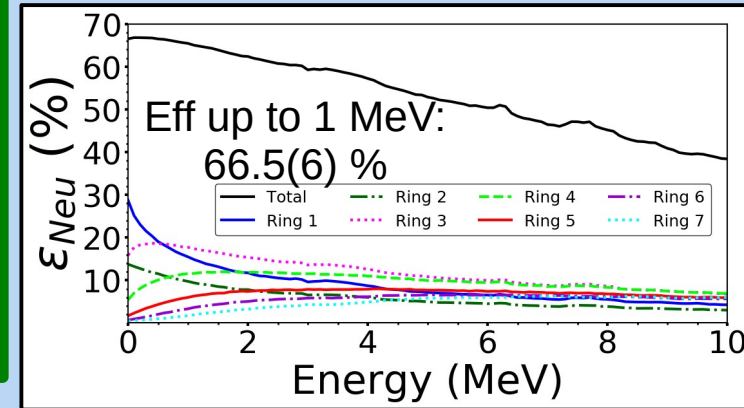
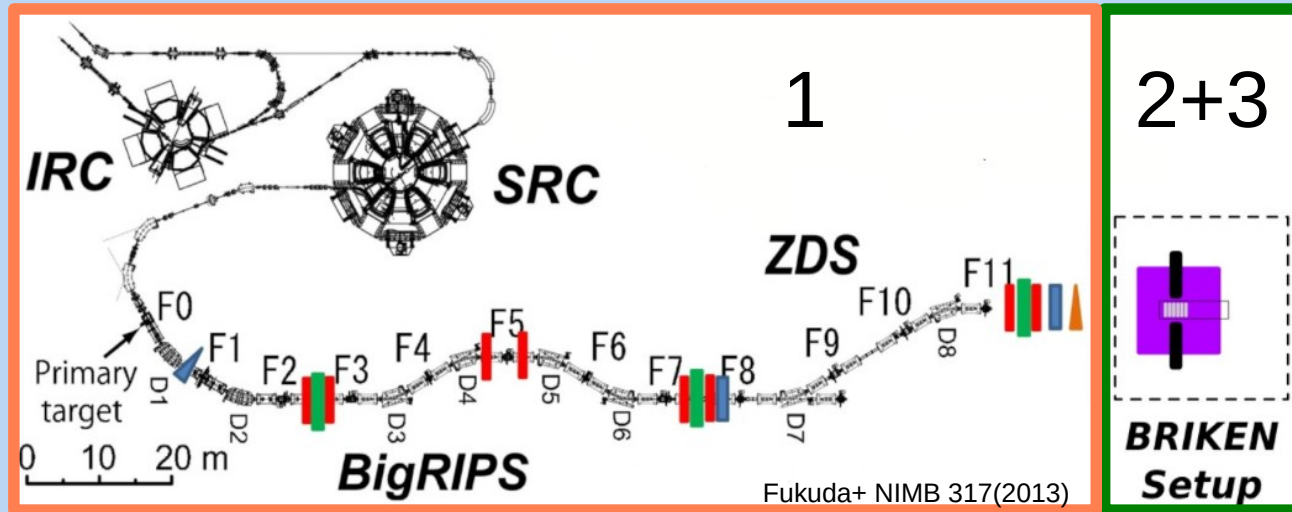
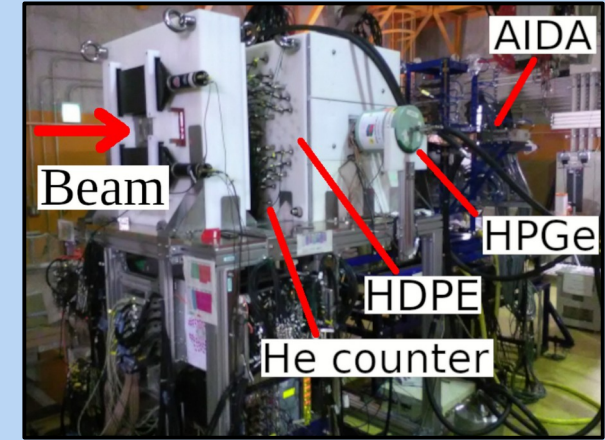
Cowan+ Physics Today (2004)



- $T_{1/2}$ and P_{xn} of very neutron-rich nuclei for $55 \leq Z \leq 64$ are key for the REP formation.

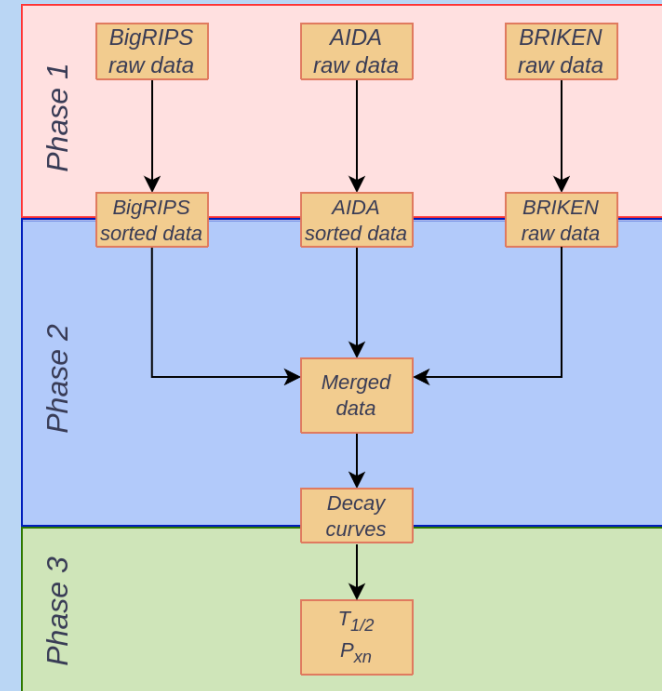
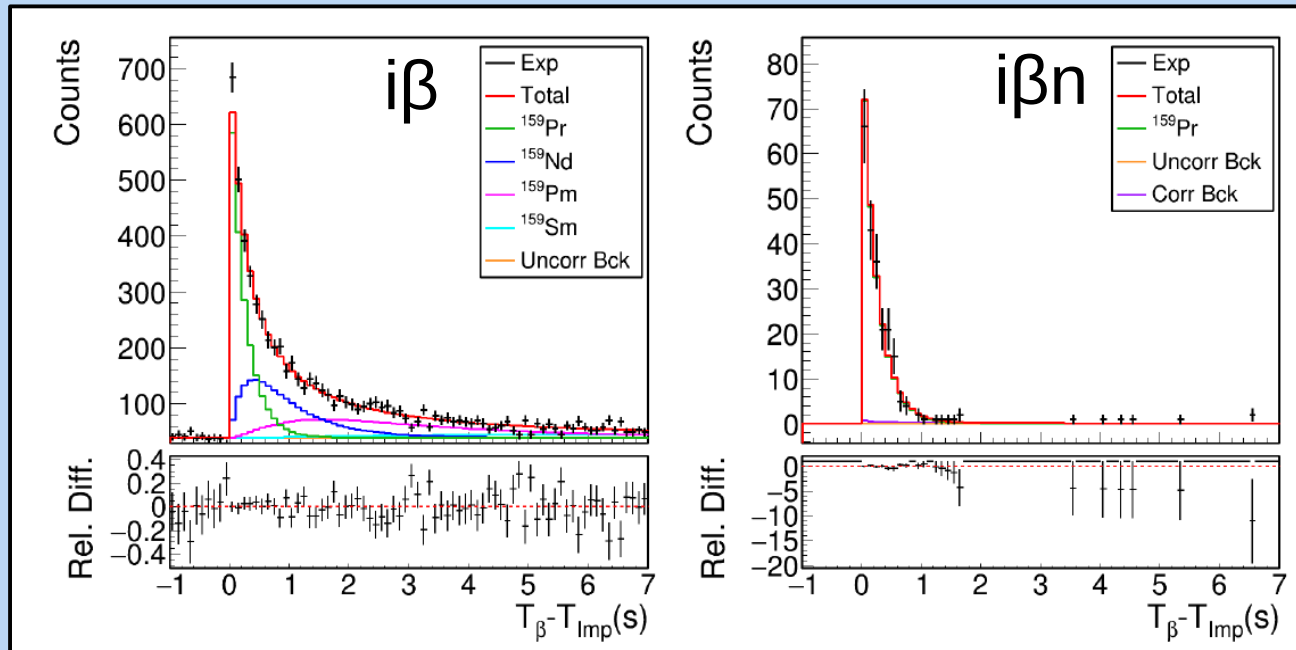
Experimental setup

1. **BigRIPS**: Production, separation and identification
2. **AIDA**: Implantation and β -decay detection.
3. **BRIKEN**: β -delayed neutron (+gamma) detection



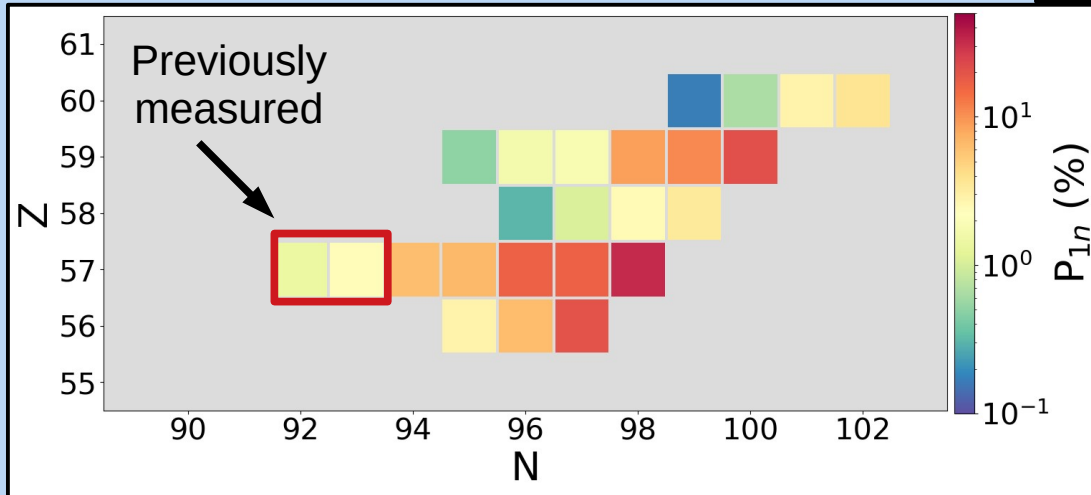
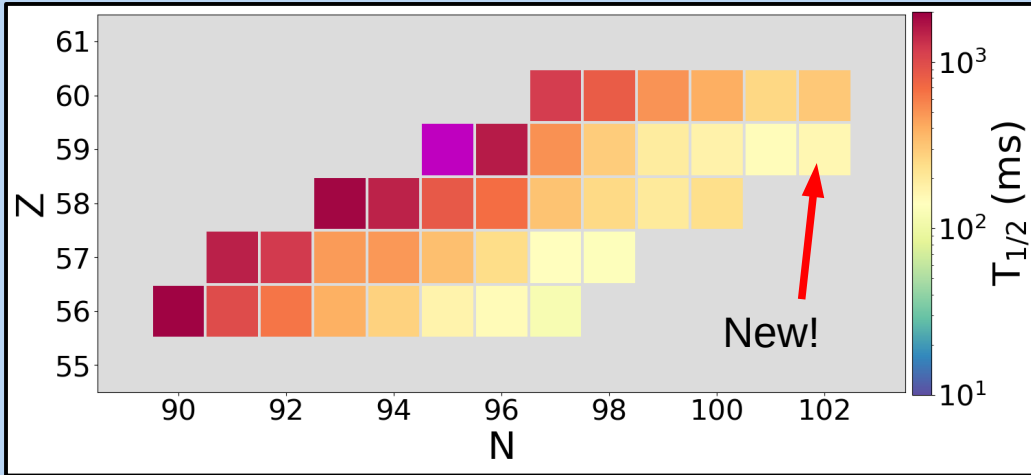
Data analysis

- **Simultaneous fit** of the coincidence curves using binned maximum likelihood.
- **Fitted parameters:** parent isotope decays ($N_1(t=0)$), its $T_{1/2}$, and its P_{1n} .



Global results

- **36 $T_{1/2}$** , including a **first-time** measurement (^{161}Pr).
- The $T_{1/2}$ reported **overall consistent** results with previous measurements, reducing the uncertainties of 13 nuclei.

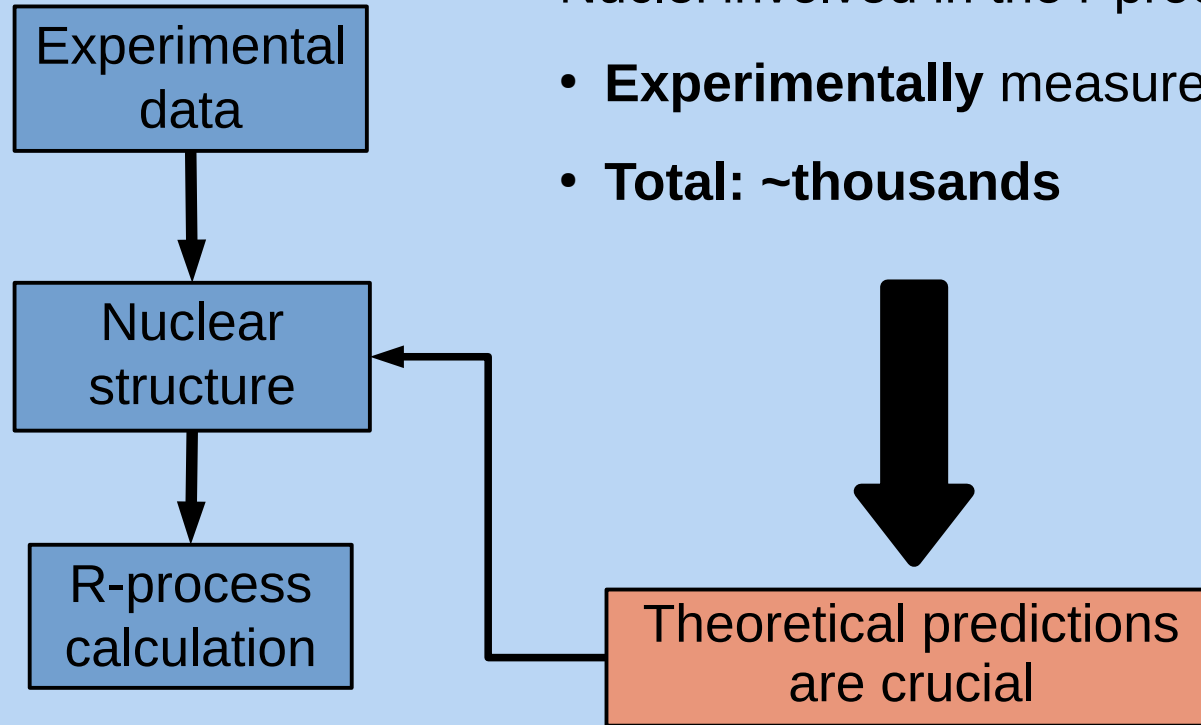


- **24 P_{1n}** values have been measured, **22** of them for the **first-time**.

R-process calculation

Nuclei involved in the r-process:

- **Experimentally** measured: ~hundreds
- **Total**: ~thousands



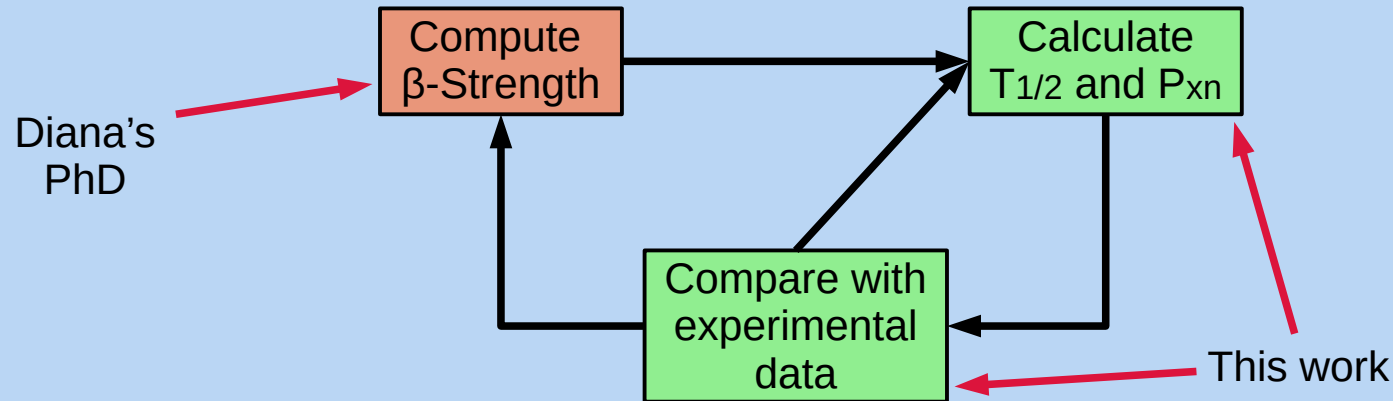
Nuclear structure calculations

In collaboration with GSI Theoretical Nuclear Astrophysics Group.

1. Compute the **β -Strength** (Diana's PhD).
2. Calculate **$T_{1/2}$ and P_{xn}** .
3. **Compared** with the experimental data from REP-BRIKEN.
4. **Refine** the model parameters.

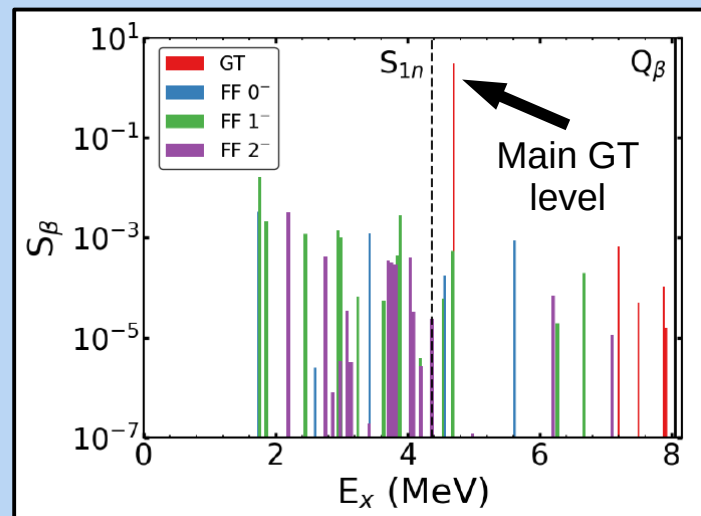


Diana Alvear Terrero



Nuclear structure calculations

- **Revision** of the Marketin et al. Model (QRPA).
- The calculation includes **GT** and **FF** transitions.
- The states are computed as **discrete energy levels**.
- P_{xn} calculated using **HF** model through TALYS.
- **FRDM** mass model to compute Q_β and S_{xn}.



$$\lambda^{(m)} = \frac{\ln 2}{K} \int_1^{W_0^{(m)}} \boxed{F(Z, W)} W p (W - W_0^{(m)})^2 \boxed{C^{(m)}(W, W_0^{(m)})} dW$$

Decay rate

Fermi function

Shape factor

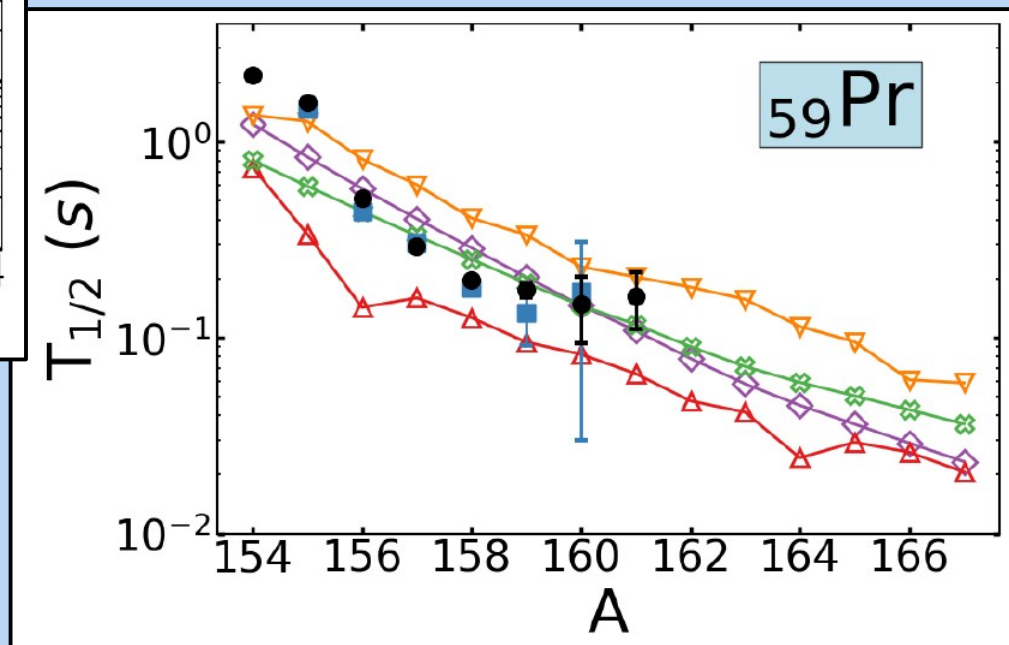
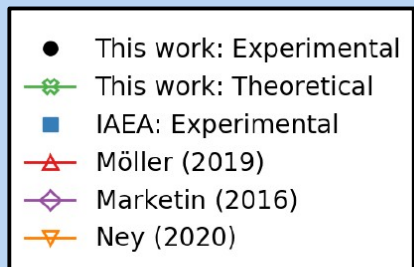
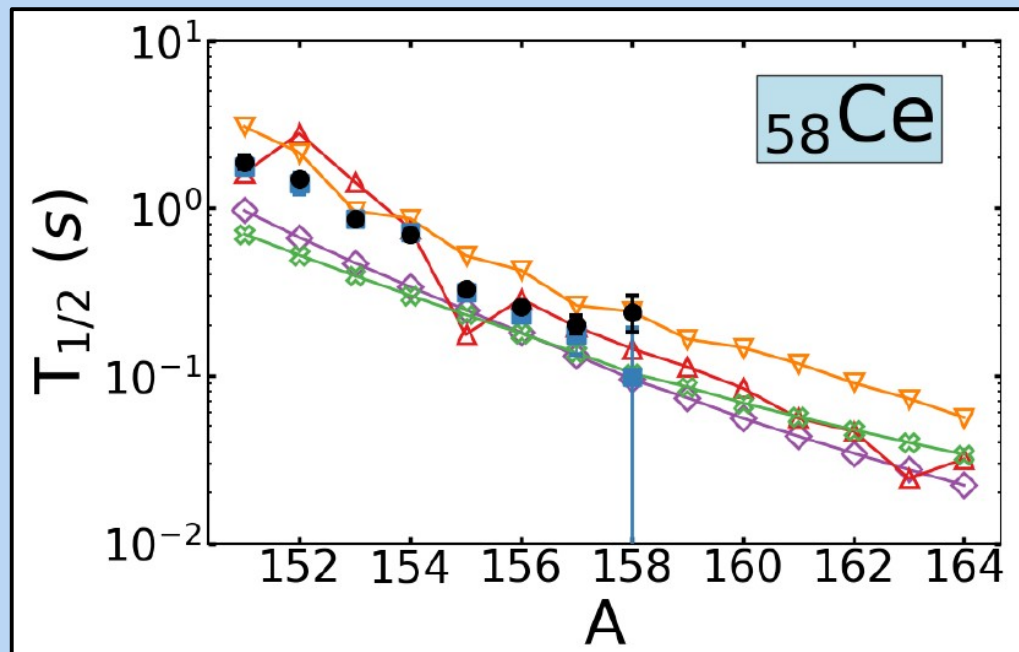
$$\lambda(E_x) = \sum_m^{E_m^* < Q_\beta} \lambda^{(m)} \delta(E_x - E_m^*)$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \int_0^{Q_\beta} \lambda(E_x) dE_x$$

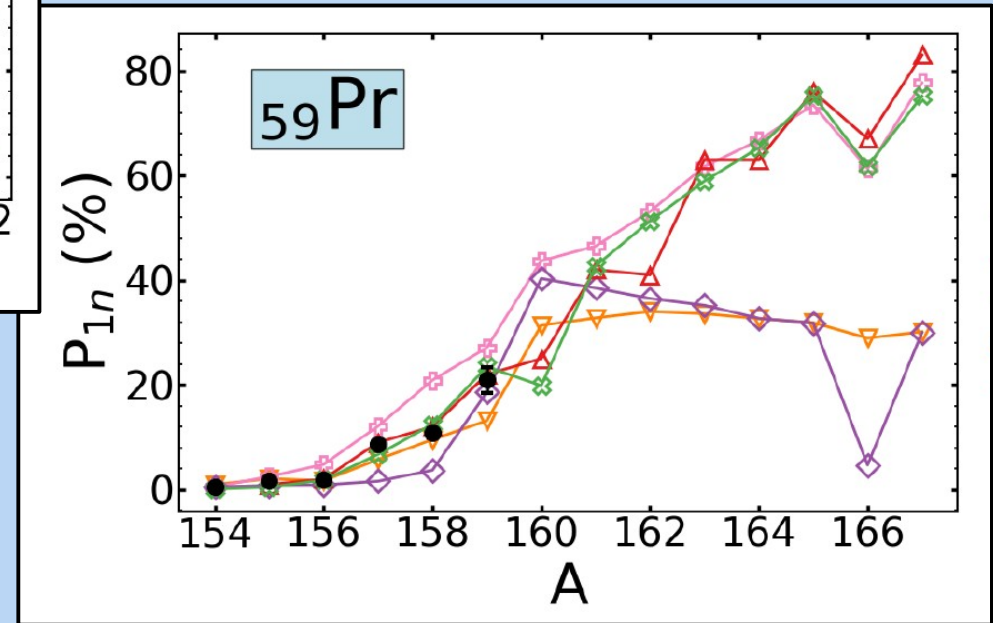
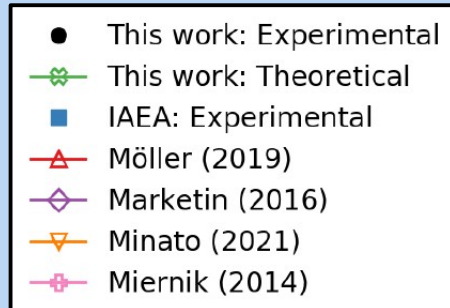
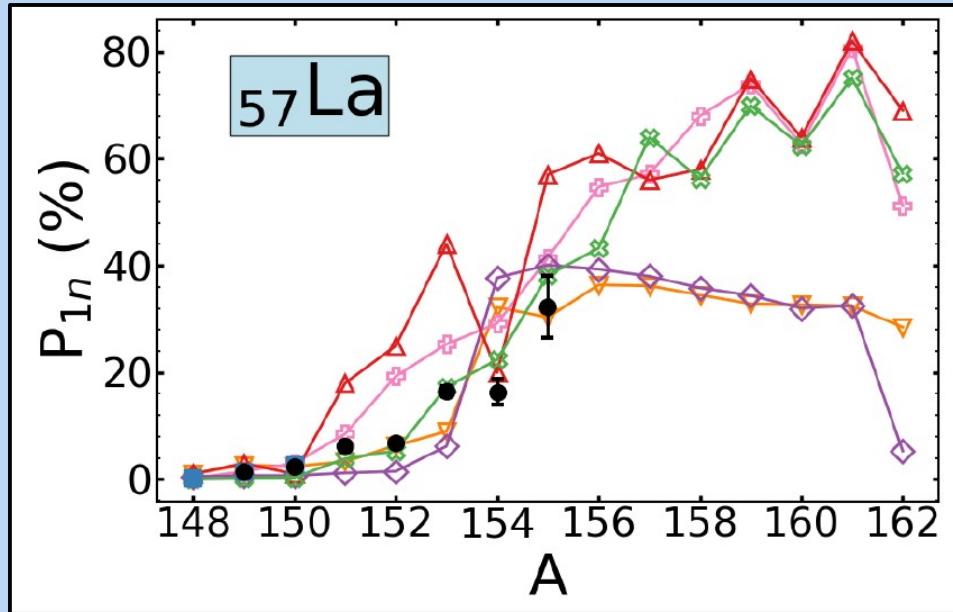
TALYS

$$P_{xn} = \int_0^{Q_\beta} \lambda(E_x) \cdot \boxed{p_{xn}(E_x)} dE_x$$

Some results $T_{1/2}$



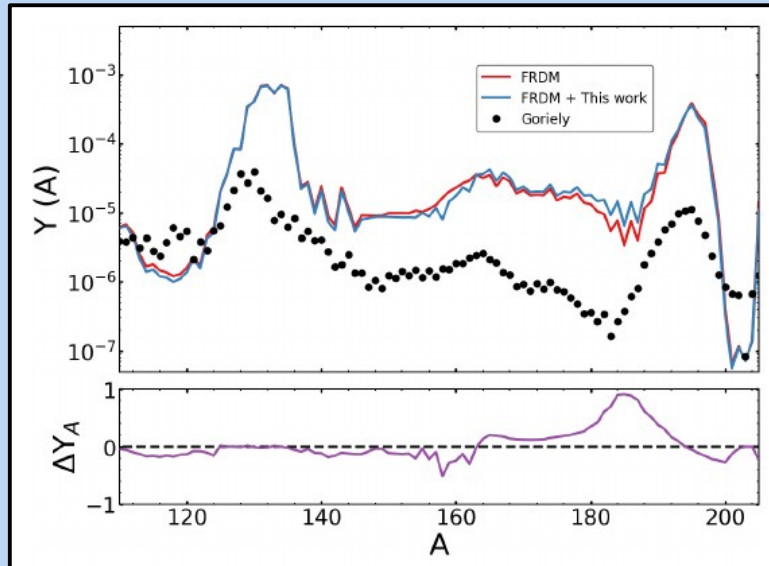
Some results P_{1n}



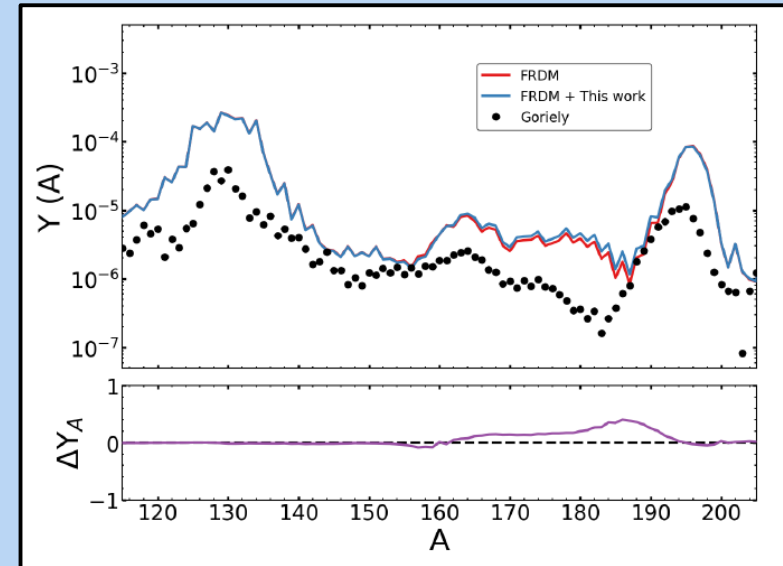
Preliminary astrophysical impact

- Input: Our new **experimental** data + refined nuclear **model** from Ba to Nd.
- Trajectories of the **dynamical ejecta** from a neutron star merger.
- The **impact depends** hugely on the **trajectories** selected.

Single trajectory



Average of multiple trajectories



Conclusions

- First comprehensive **characterization** of the BRIKEN neutron counter.
- The analysis of the **REP-BRIKEN experiment** was successful, yielding:
 - ▷ **36 $T_{1/2}$** , of which 13 with improved precision and **1 first-time** measure.
 - ▷ **24 P_{1n}** , of which **22 are first-time** measurements.
- We updated and benchmarked a **QRPA nuclear structure model** using our new experimental data. This new model predicted with **good accuracy the P_{1n}** of the odd-Z nuclei studied in this thesis.
- We presented a first overview of the **astrophysical impact** of both the experimental and theoretical new data on the r-process final abundances.

Acknowledgments

Collaborators

A. Tarifeño-Saldivia¹, A. Tolosa-Delgado^{2,1}, D. Alvear-Terrero^{3,4}, G. Martínez-Pinedo^{3,4,5}, Z. Xiong⁵, G. G. Kiss⁶, J. L. Tain², A. Vitéz-Sveicz⁶, BRIKEN collaboration⁹

¹ Instituto de Física Corpuscular (CSIC-UV), Spain. ² European Organization for Nuclear Research (CERN), Switzerland. ³ Institut für Kernphysik (TU Darmstadt), Germany. ⁴ Helmholtz Forschungsakademie Hessen für FAIR (GSI), Germany. ⁵ GSI Helmholtzzentrum für Schwerionenforschung, Germany. ⁶ Institute for Nuclear Research (Atomki), Hungary. ⁷ www.wiki.edu.ac.uk/display/BRIKEN/Home.

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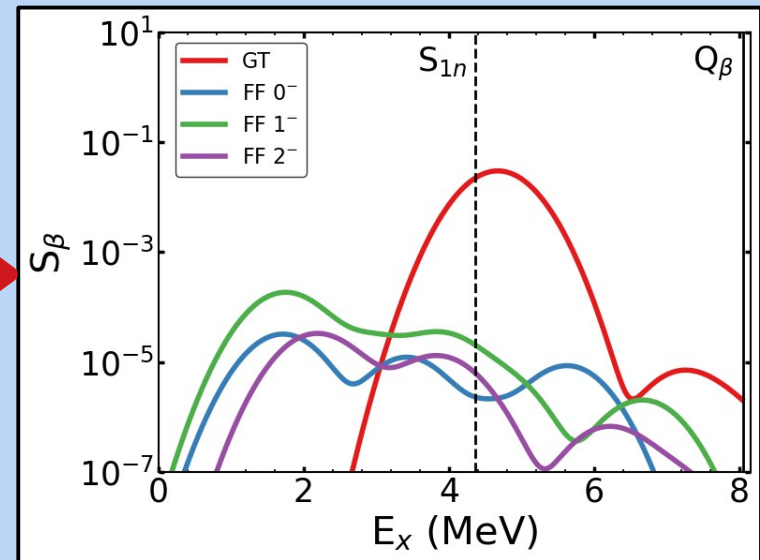
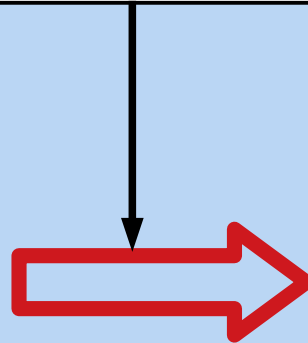
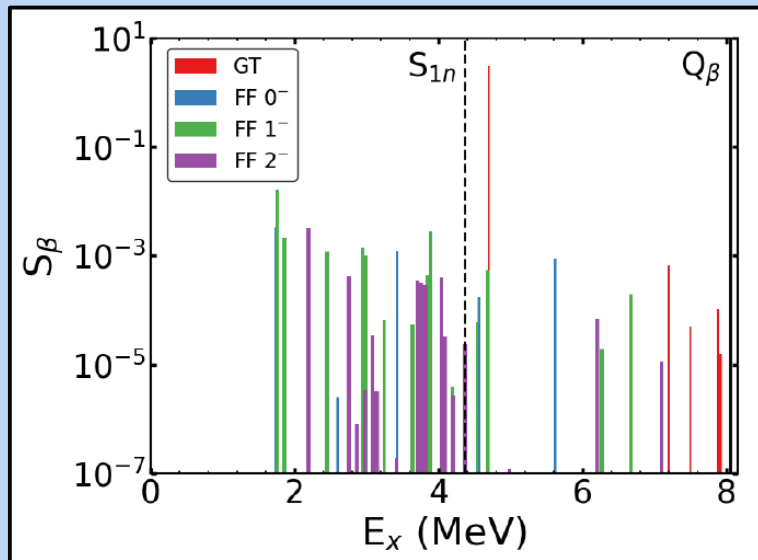


Neutron spectrometry

Smearing function

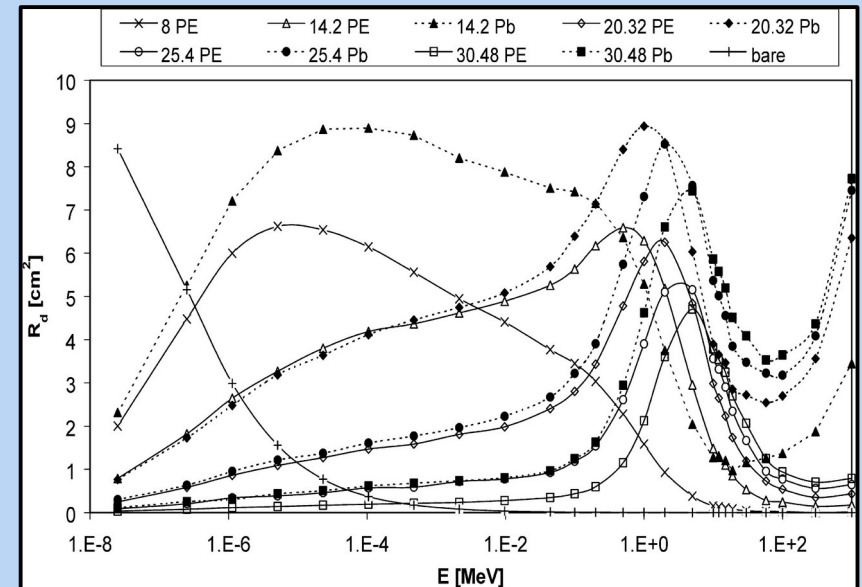
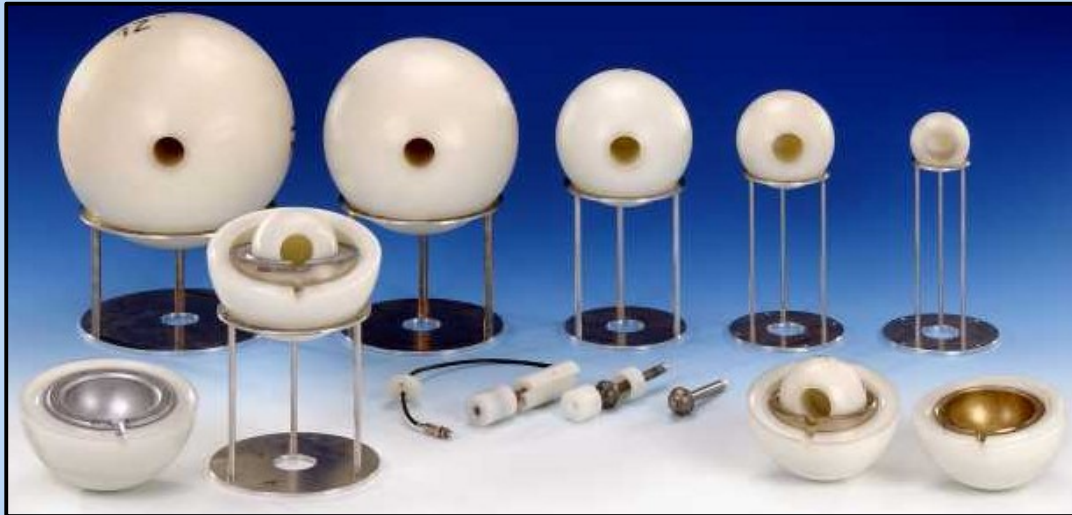
- The QRPA calculation only computes **some** of the excited levels.
- To account for **higher-order terms**, we add a **smearing function** to obtain a smoother distribution.

$$G_{\sigma}(W, W_0) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(W - W_0)^2 m_e^2}{2\sigma^2}\right)$$



Neutron spectrometry

- Using different moderator designs, one can detect neutrons of different energies.
- The detection mechanism is based on neutron thermalization (bonner spheres).
- Energy sensitivity from meV to MeV (even GeV).

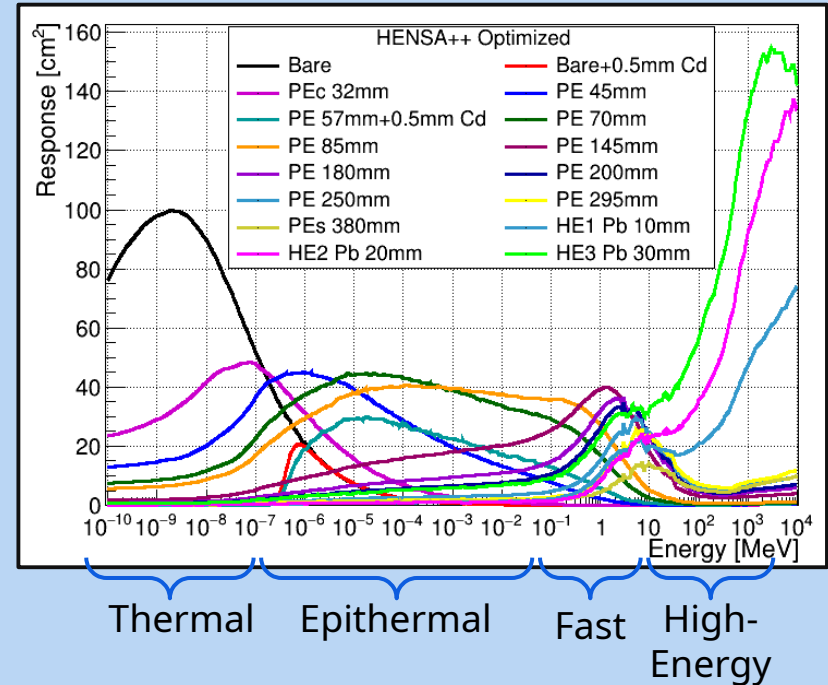


HENSA project

- Rectangular geometry.
- Higher efficiency (larger ^3He detectors).
- Scalability: Simplified setup
- Wider Energy Range: up to the GeV.

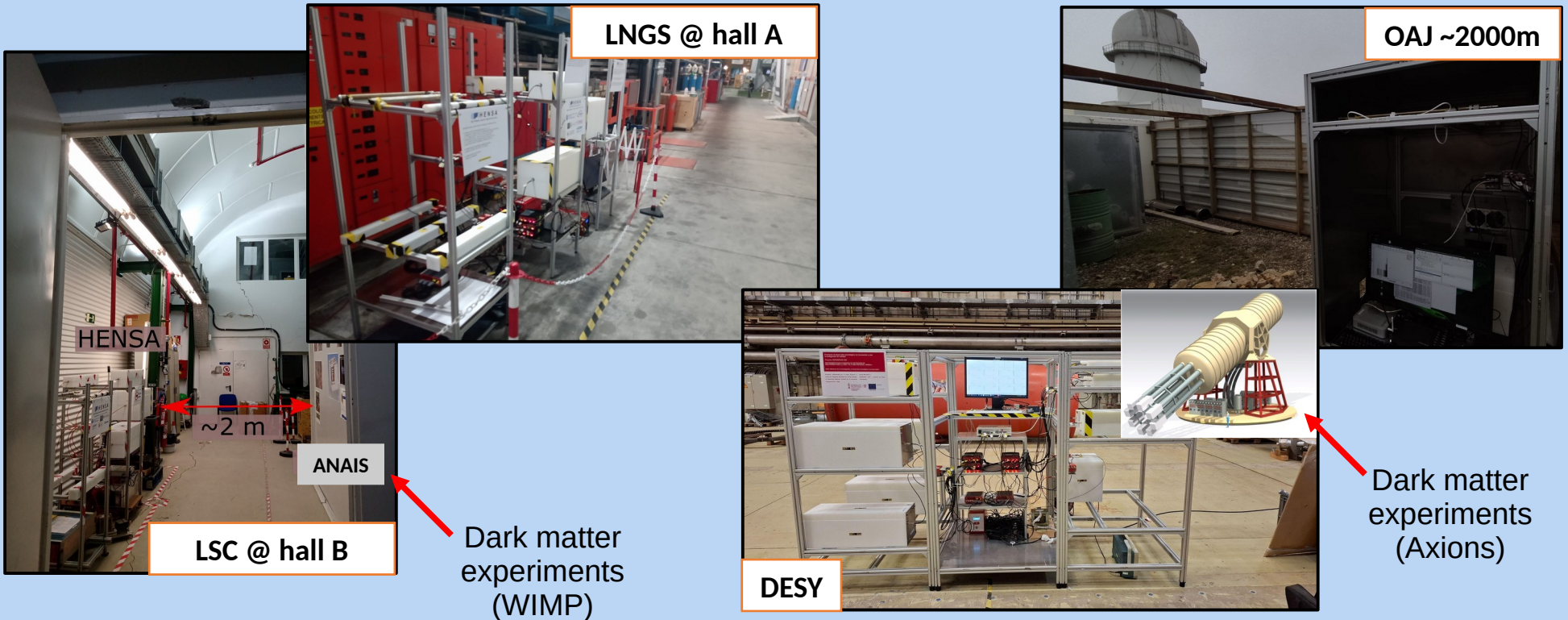


High Efficiency Neutron Spectrometry Array



HENSA uses

- Underground labs' background characterization (LSC, LNGS, Felsenkeller)
- Cosmic-ray neutron spectrum + space weather (OAJ, DESY)



Unfolding

Unfolding consists on extracting the **flux magnitude and energy distribution** from the measured **rates** and the **response**.

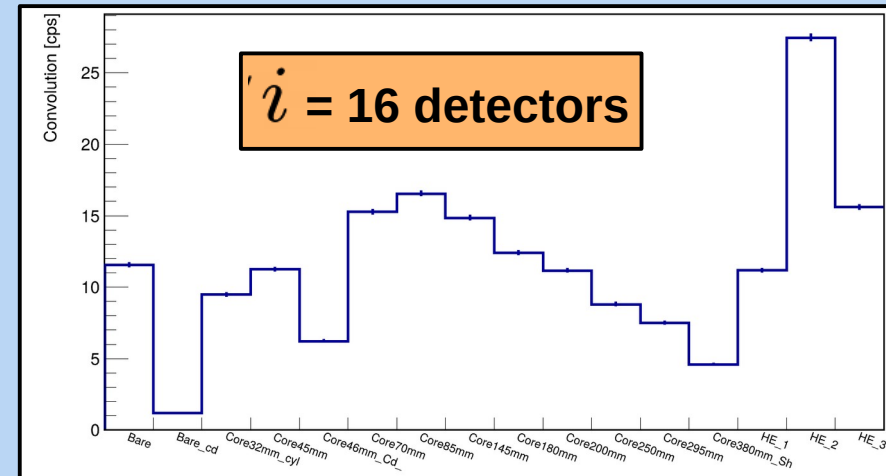
$$C_i = \sum_{j=1}^m R_{ij} \phi_j \quad i = 1, 2, \dots, n$$

Diagram illustrating the unfolding equation:

- C_i : Detector **counting rate** (measurement)
- R_{ij} : Detector **response** (calculated)
- ϕ_j : Neutron **flux** (magnitude we want to determine)

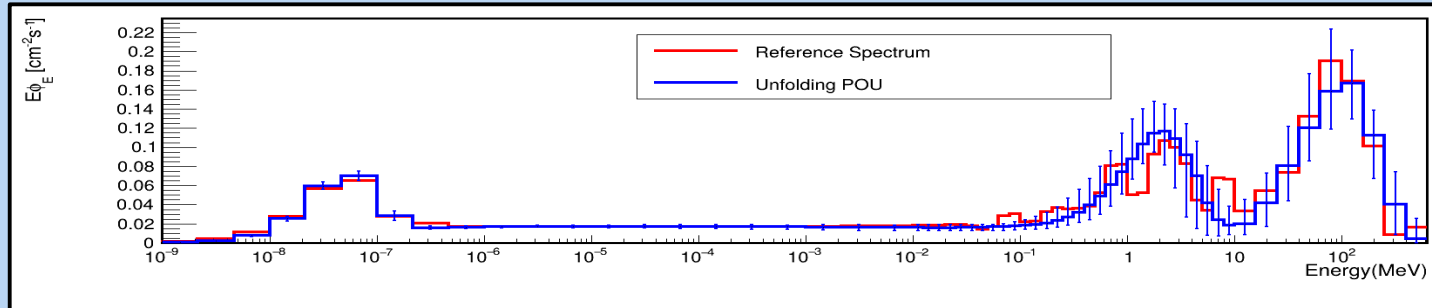
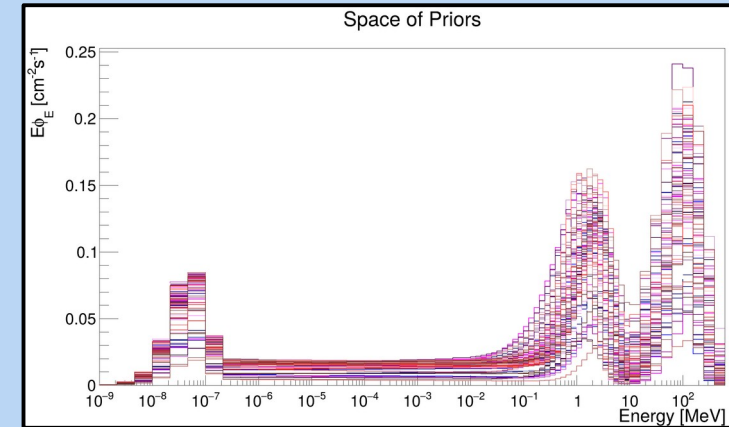
Linear inverse problem (n < m)!!!

The unfolding methods are based on the use of a priori information.



POU

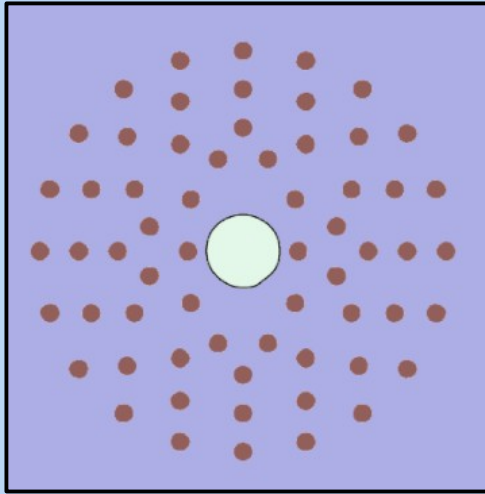
- Many different unfolding algorithms exist (MITOM, FRUIT, MAXED, GRAVEL, ...), but none have a standardized uncertainty evaluation.
- Our code is called POU (Prior Optimizer for Unfolding).
 - Use of a parametric space for the prior.
 - The output solution is a combination of accepted solutions. -----> Systematic assessment of the errors.



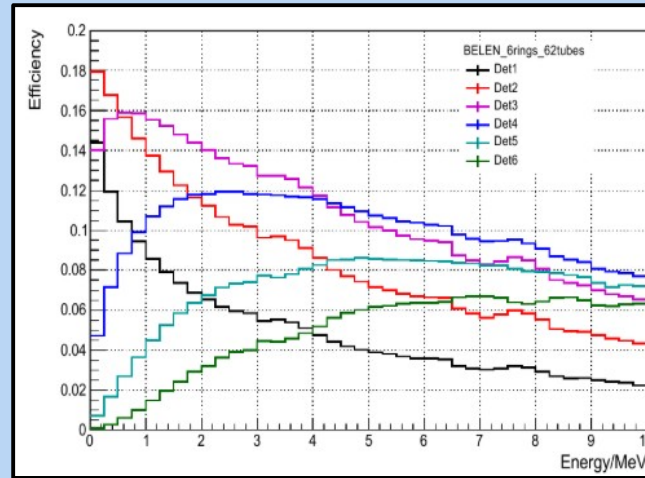
A. Quero et al,
POU: A novel parametric-iterative
unfolding algorithm for Bonner
Sphere Spectrometry.
Accepted RPC

Extra: Future BRIKEN?

Low-resolution neutron spectrometer for beta-delayed neutron experiments

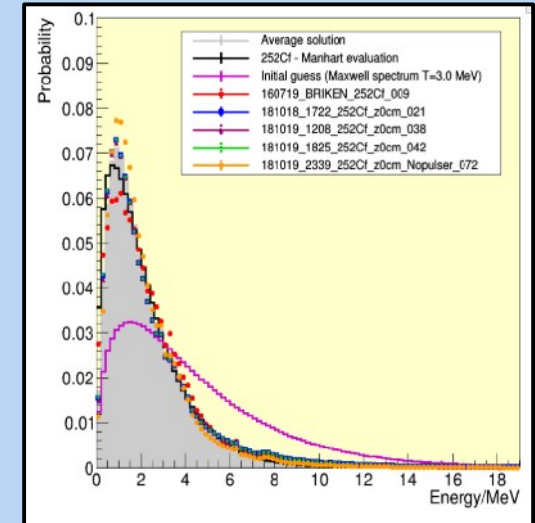


Different spatial
positions

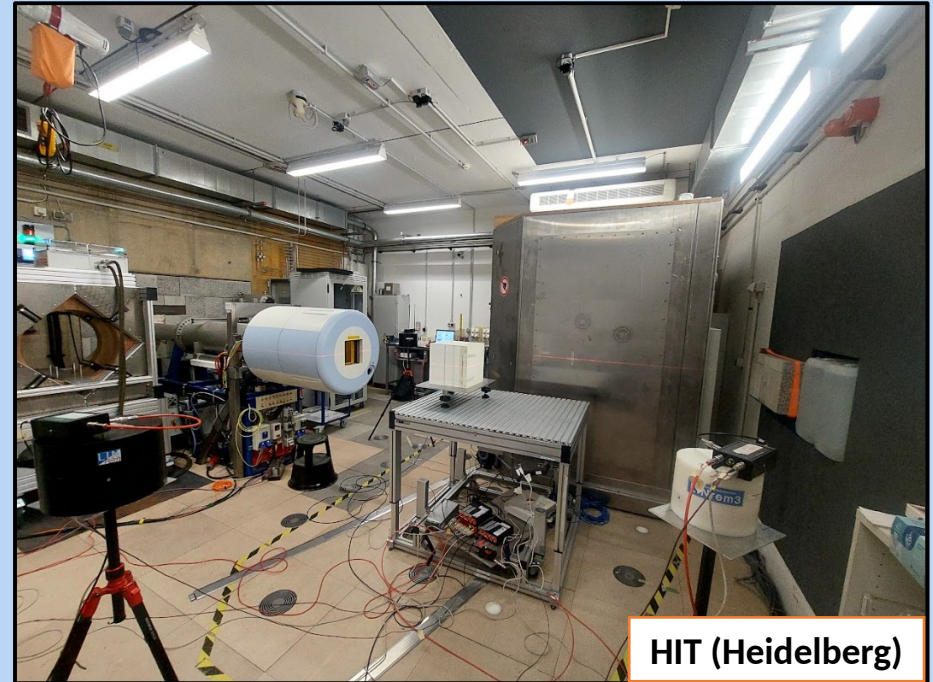


Different energy
response

**Conceptual test
with 252Cf**

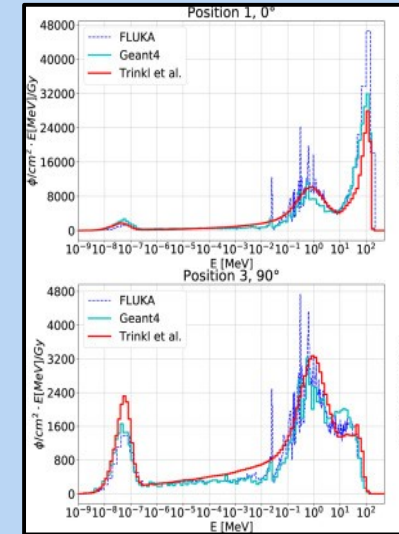
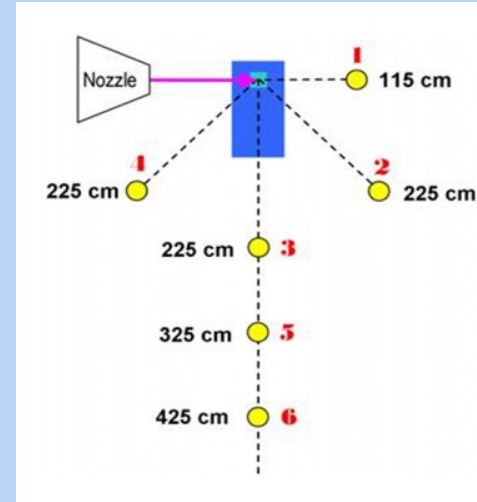
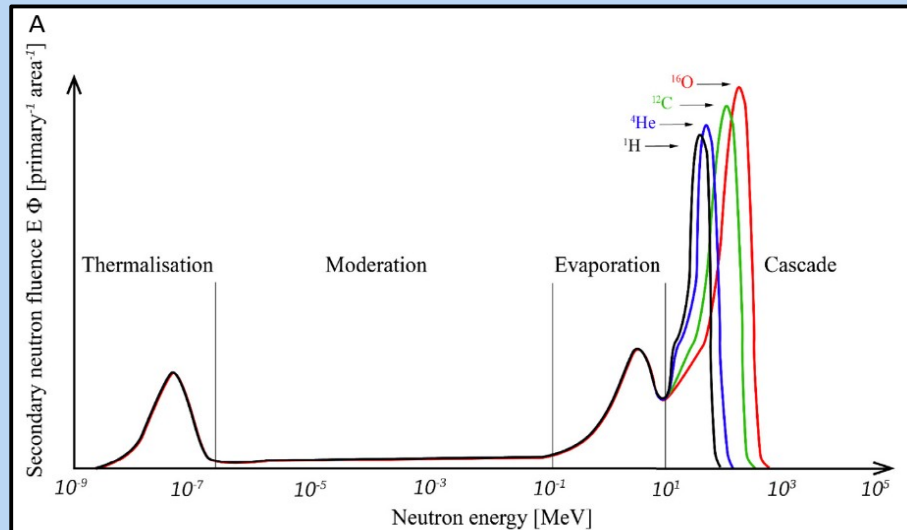


Neutron dosimetry and very high flux environments



Hadrontherapy and neutrons

Stray or secondary neutrons produced during hadrontherapy pose a potential risk for long-term cancer, especially in **pediatric patients**, and evaluation of effect in **pregnant women**.



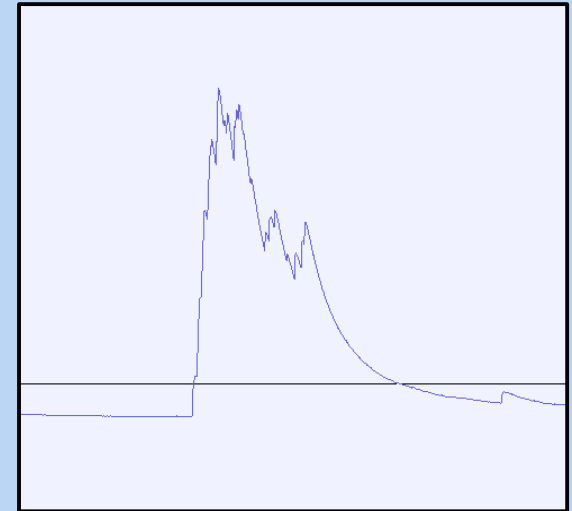
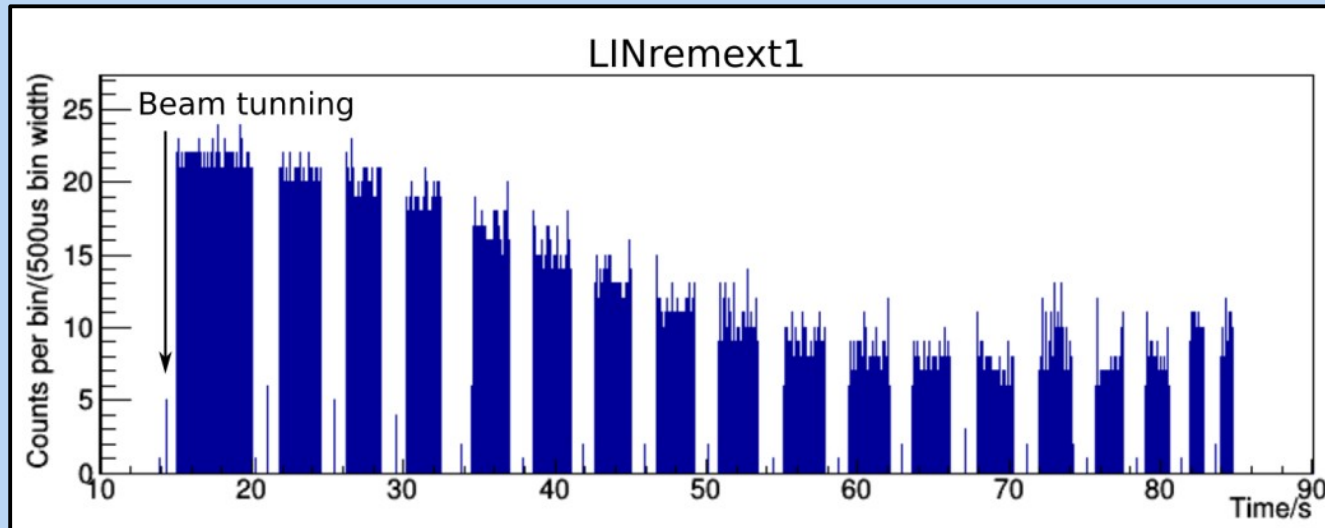
Some challenges:

- **Angular** dependence
- **Wide energies range**
- **Projectile** dependence
- Dependent on the **technology**
- **Very high flux**

Data stream in hadrontherapy

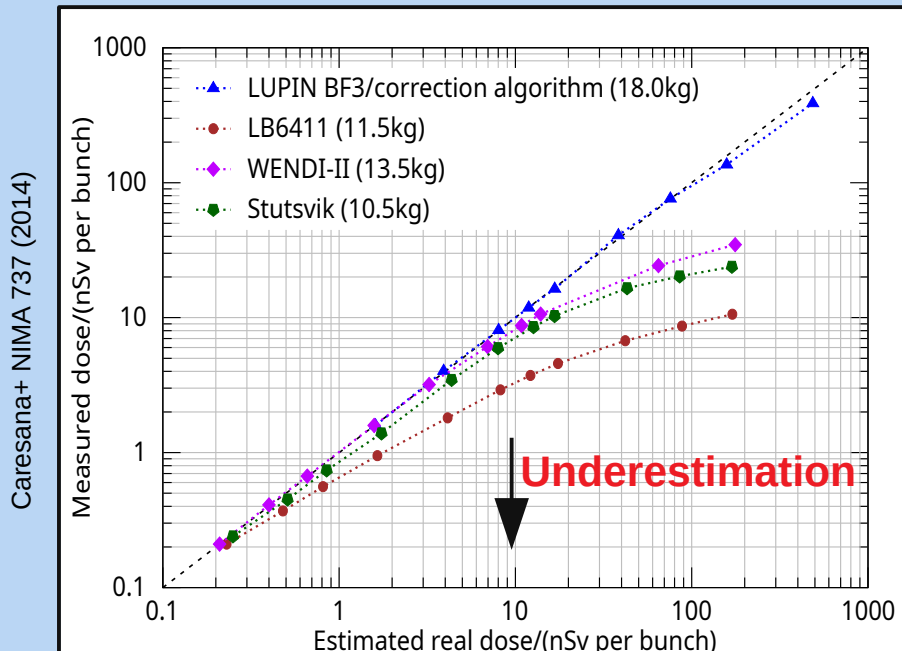
- Pulsed fields
- Very high instantaneous rate
- Very high gamma and neutron secondary production
- Possible charged particles reach your detector

**Pileup
and
dead time**



Comercial dosimeter

Most of the commercial dosimeter are not suited for pulsed neutron fields (pile-up effect). **They present underestimation of the dose beyond 10 nSv/burst.**



LINrem: Ambient neutron dosimeter

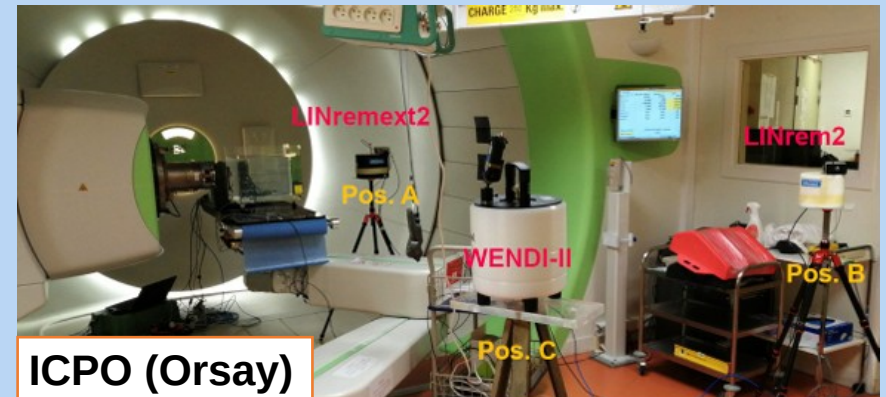
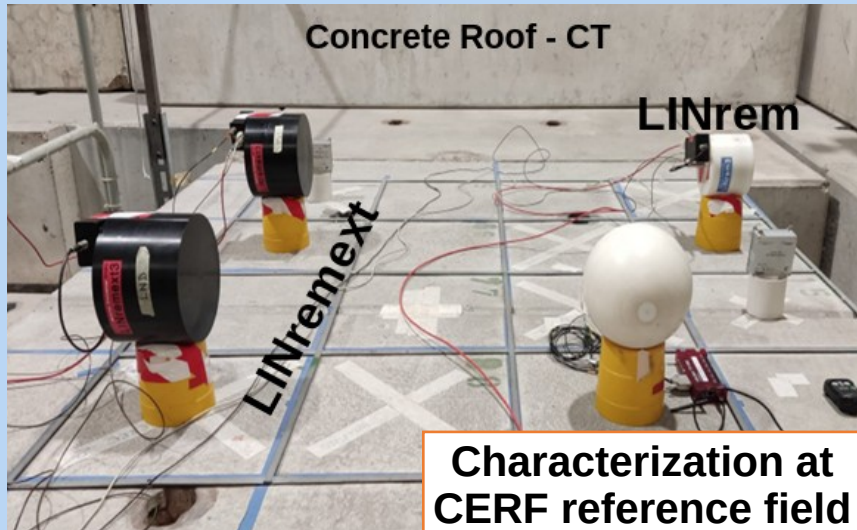
- Precise response with “light” weight.
- DAQ suited for complex radiation fields.
- Digital acquisition and readout.
- Compatible with ICRU95 (new standard)
- Very low gamma efficiency



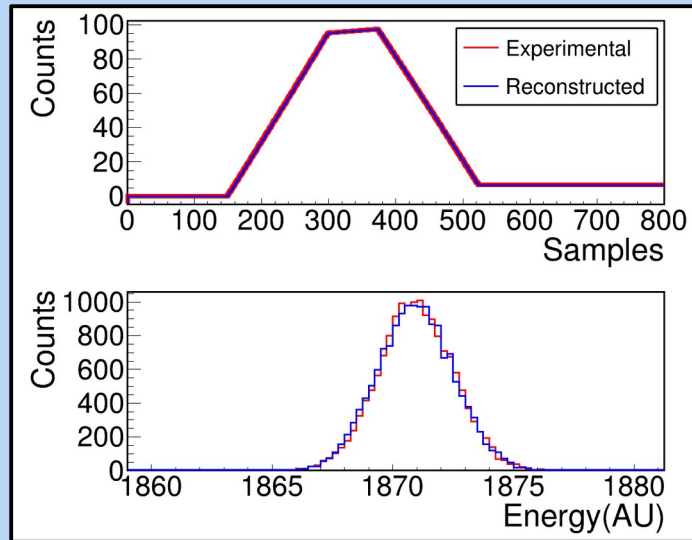
Patents granted in the US, EU, JP

TRL6 → Proved that work in a relevant environment

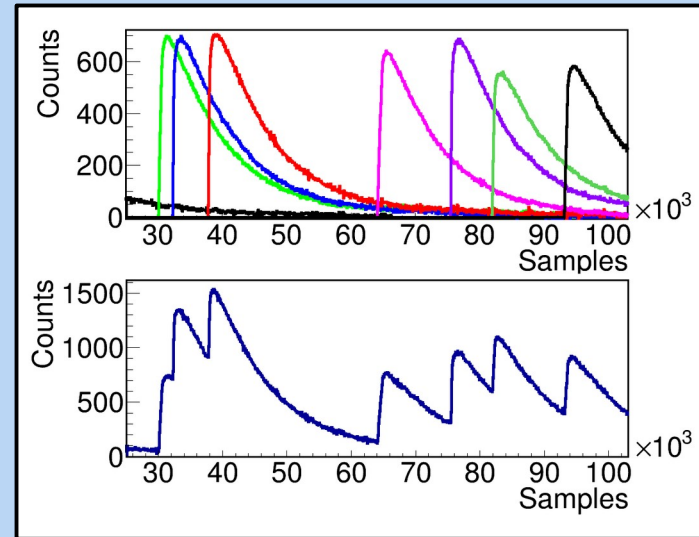
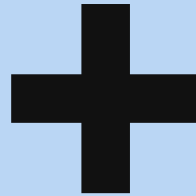
&
passive



LINrem software solution

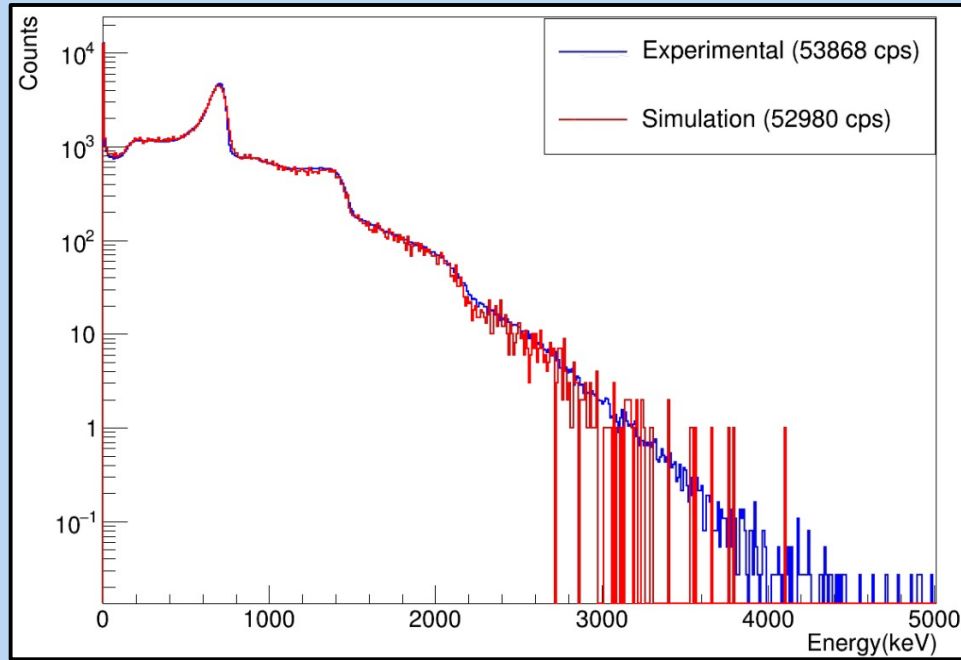


**DAQ firmware
emulation**



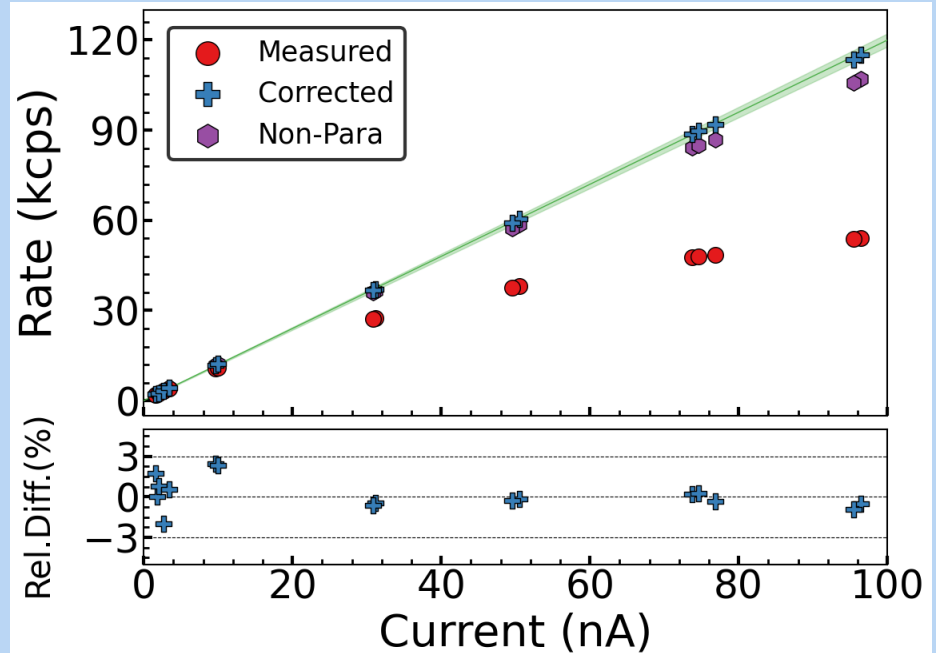
**Data stream
simulation**

LINrem solution



**Accurate energy
simulation**

Recover linearity



FEBEX?

STRUCK SIS3316

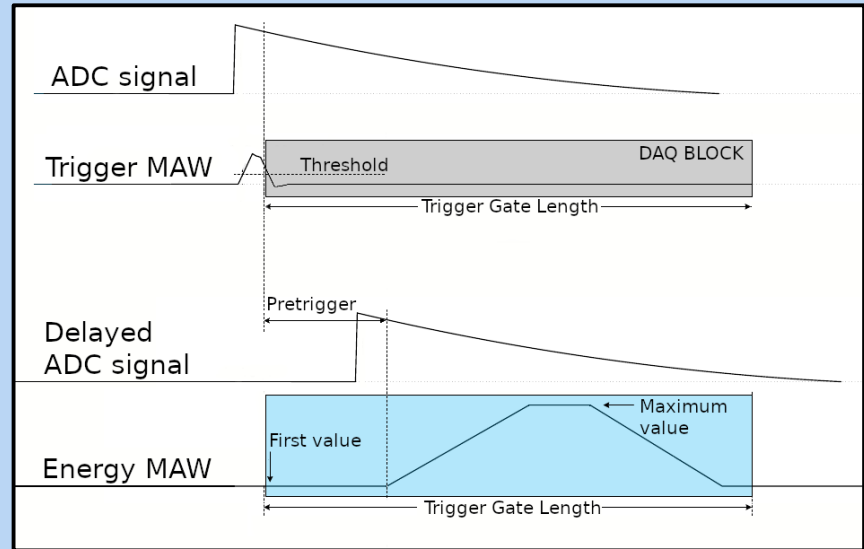
16 ch

14 bit

250MHz



Same structure as FEBEX



Thank you!

How were the elements from the universe made?

1

1

H

Hydrogen

1.008

1

2

IIA

4

Be

Beryllium

9.013

2

3

Li

Lithium

6.94

3

11

Na

Sodium

22.98

11

19

K

Potassium

39.10

19

37

Rb

Rubidium

85.47

37

55

Cs

Cesium

132.91

55

87

Fr

Francium

[223]

87

13

IIIA

5

B

Boron

10.81

3

14

IVA

6

C

Carbon

12.011

4

15

VA

7

N

Nitrogen

14.007

5

16

VIA

8

O

Oxygen

15.999

6

17

VIIA

9

F

Fluorine

18.998

7

13

Al

Aluminum

26.98

13

14

Si

Silicon

28.085

14

15

P

Phosphorus

30.97

15

16

S

Sulfur

32.06

16

17

Cl

Chlorine

35.45

17

18

Ar

Argon

39.95

18

21

Sc

Scandium

44.96

21

23

V

Vanadium

50.94

23

25

Mn

Manganese

54.94

25

27

Co

Cobalt

58.93

27

29

Cu

Copper

63.55

29

31

Ga

Gallium

69.72

31

33

As

Arsenic

74.92

33

35

Br

Bromine

79.90

35

37

Kr

Krypton

83.80

36

39

Y

Yttrium

88.91

39

41

Nb

Niobium

92.91

41

43

Tc

Technetium

[98]

43

45

Rh

Rhodium

102.91

45

47

Pd

Palladium

106.42

46

49

In

Indium

114.82

49

51

Sb

Antimony

121.76

51

53

I

Iodine

126.90

53

55

Xe

Xenon

131.29

54

57-71

Ln

Lanthanides

72

Hf

Hafnium

178.49

72

73

Ta

Tantalum

180.95

73

74

W

Tungsten

183.84

74

75

Re

Rhenium

186.21

75

76

Os

Osmium

190.23

76

77

Ir

Iridium

192.22

77

78

Pt

Platinum

195.08

78

79

Au

Gold

196.97

79

80

Hg

Mercury

200.59

80

81

Tl

Thallium

204.38

81

82

Pb

Lead

207.2

82

83

Bi

Bismuth

208.98

83

84

Po

Polonium

[209]

84

85

At

Astatine

[210]

85

86

Rn

Radon

[222]

86

89-103

Ac

Actinides

104

Rf

Rutherfordium

[261]

104

105

Db

Dubnium

[262]

105

106

Sg

Seaborgium

[266]

106

107

Bh

Bohrium

[264]

107

108

Hs

Hassium

[277]

108

109

Mt

Meitnerium

[268]

109

110

Ds

Darmstadtium

[271]

110

111

Rg

Roentgenium

[272]

111

112

Cn

Copernicium

[285]

112

113

Nh

Nihonium

[284]

113

114

Fl

Flerovium

[289]

114

115

Mc

Moscovium

[290]

115

116

Lv

Livermorium

[293]

116

117

Ts

Tennessine

[294]

117

118

Og

Oganesson

[294]

118

57

La

Lanthanum

138.91

57

58

Ce

Cerium

140.12

58

59

Pr

Praseodymium

140.91

59

60

Nd

Neodymium

144.24

60

61

Pm

Promethium

[145]

61

62

Sm

Samarium

150.36

62

63

Eu

Europium

151.96

63

64

Gd

Gadolinium

157.25

64

65

Tb

Terbium

158.93

65

66

Dy

Dysprosium

162.50

66

67

Ho

Holmium

164.93

67

68

Er

Erbium

167.26

68

69

Tm

Thulium

168.93

69

70

Yb

Ytterbium

173.05

70

71

Lu

Lutetium

174.97

71

89

Ac

Actinium

[227]

89

90

Th

Thorium

232.04

90

91

Pa

Protactinium

231.04

91

92

U

Uranium

238.03

92

93

Np

Neptunium

[237]

93

94

Pu

Plutonium

[244]

94

95

Am

Americium

[243]

95

96

Cm

Curium

[247]

96

97

Bk

Berkelium

[247]

97

98

Cf

Californium

[251]

98

99

Es

Einsteinium

[252]

99

100

Fm

Fermium

[257]

100

101

Md

Mendelevium

[258]

101

102

No

Nobelium

[259]

102

103

Lw

Lawrencium

[260]

103

18

VIIIA

2

He

Helium

4.0026

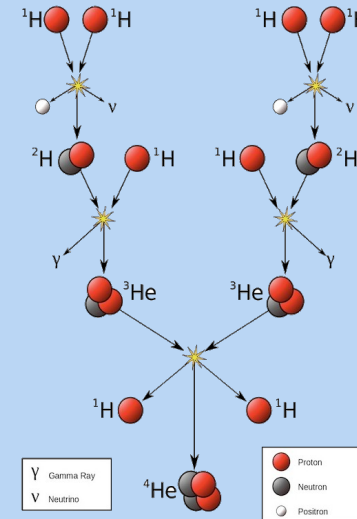
2

Early universe

+Z

- 

1 IA																											18 VIIIA		
1 H Hydrogen 1.008																											2 He Helium 4.0026		
2 IIA																													
3 Li Lithium 6.94	4 Be Beryllium 9.013																	5 B Boron 10.81		6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.18					
2-1																		2-3		2-4	2-5	2-6	2-7	2-8					
11 Na Sodium 22.98	12 Mg Magnesium 24.32																	13 Al Aluminum 26.98		14 Si Silicon 28.085	15 P Phosphorus 30.97	16 S Sulfur 32.06	17 Cl Chlorine 35.45	18 Ar Argon 39.95					
2-8-1																		2-8-3		2-8-4	2-8-5	2-8-6	2-8-7	2-8-8					
3 IIIB		4 IVB		5 VB		6 VIB		7 VIIB		8 VIIIB		9 VIIIB		10 VIIIB		11 IB		12 IIB											
19 K Potassium 39.10	20 Ca Calcium 40.08	21 Sc Scandium 44.96	22 Ti Titanium 47.87	23 V Vanadium 50.94	24 Cr Chromium 51.996	25 Mn Manganese 54.94	26 Fe Iron 55.84	27 Co Cobalt 58.93	28 Ni Nickel 58.69	29 Cu Copper 63.55	30 Zn Zinc 65.38	31 Ga Gallium 69.72	32 Ge Germanium 72.64	33 As Arsenic 74.92	34 Se Selenium 78.96	35 Br Bromine 79.90	36 Kr Krypton 83.80												
2-8-8-1		2-8-9-2		2-8-10-2		2-8-11-2		2-8-13-1		2-8-13-2		2-8-14-2		2-8-15-2		2-8-16-2		2-8-17-2											
37 Rb Rubidium 85.47	38 Sr Strontium 87.62	39 Y Yttrium 88.91	40 Zr Zirconium 91.22	41 Nb Niobium 92.91	42 Mo Molybdenum 95.94	43 Tc Technetium 98	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.91	46 Pd Palladium 106.42	47 Ag Silver 107.87	48 Cd Cadmium 112.41	49 In Indium 114.82	50 Sn Tin 118.71	51 Sb Antimony 121.76	52 Te Tellurium 127.60	53 I Iodine 126.91	54 Xe Xenon 131.29												
2-8-18-1		2-8-18-2		2-8-18-3		2-8-18-4		2-8-18-5		2-8-18-6		2-8-18-7		2-8-18-8		2-8-18-9		2-8-18-10											



And heavy nuclei?

In 1957, the B²FH and Cameron papers proposed a solution for the formation of heavy elements:

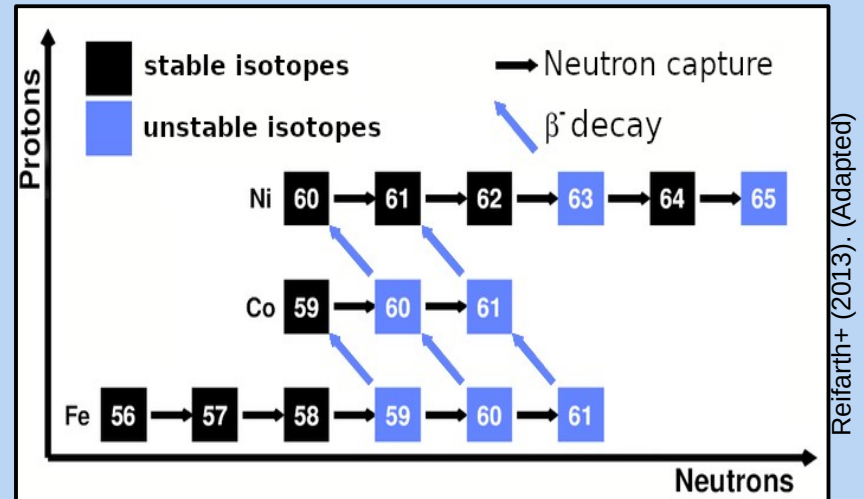
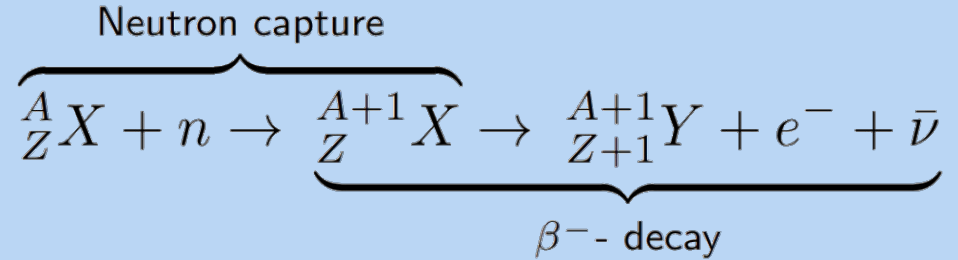
Neutron capture (n,γ)

Tends to instability

β⁻-decay

Tends to estability

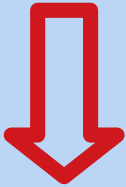
Z increase



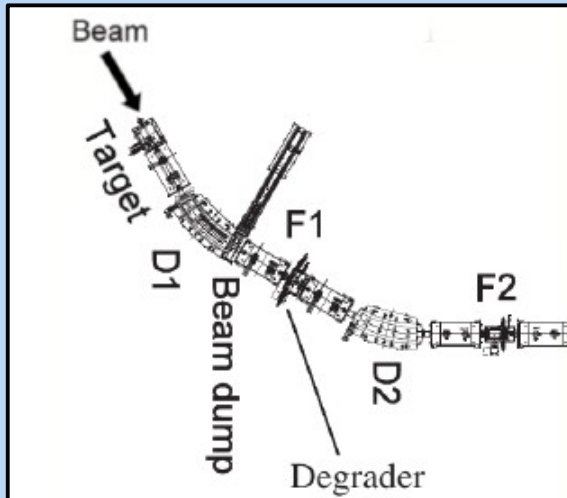
BigRIPS

Production

^{238}U beam + Be target

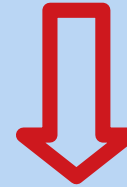


In-flight fission



Identification

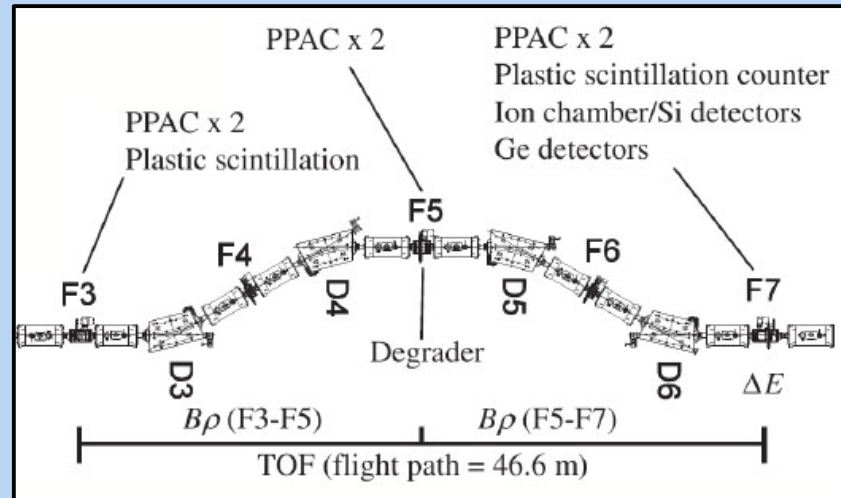
ToF-B ρ - ΔE method



Isotope identification

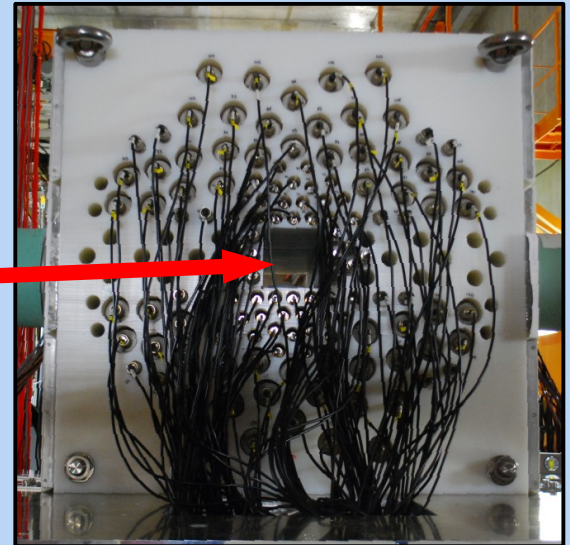
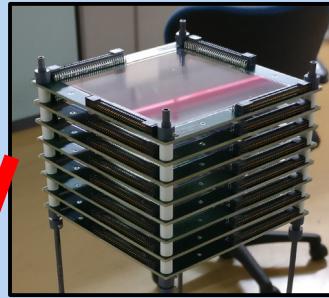
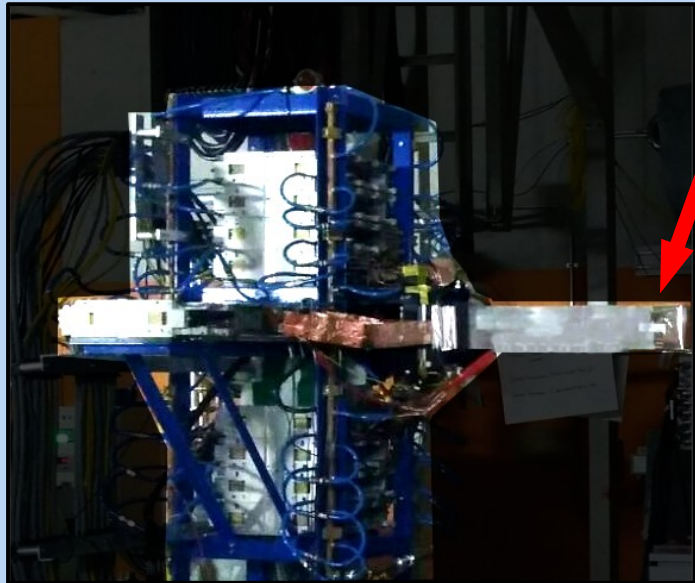
Kubo+ PTEP (2012)

Fukuda+ NIMB 317(2013)



Advanced implantation array: AIDA

- Detect **implanted ions** and **β -decays**.
- Stack of 6 Doubled-sided Silicon Strip Detectors (DSSDs).
- Each side contains 128 strips ---> **Position reconstruction**.
- Allows the detection of Implanted ions (**GeV**) and β -decay (tenths **MeV**).



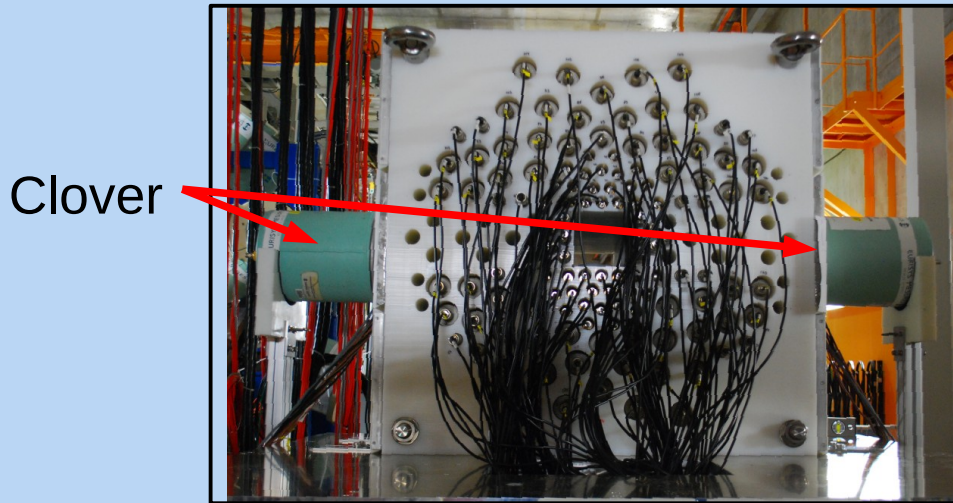
Hall+ NIMA 1050 (2023)

Griffin PhD Thesis (2019)

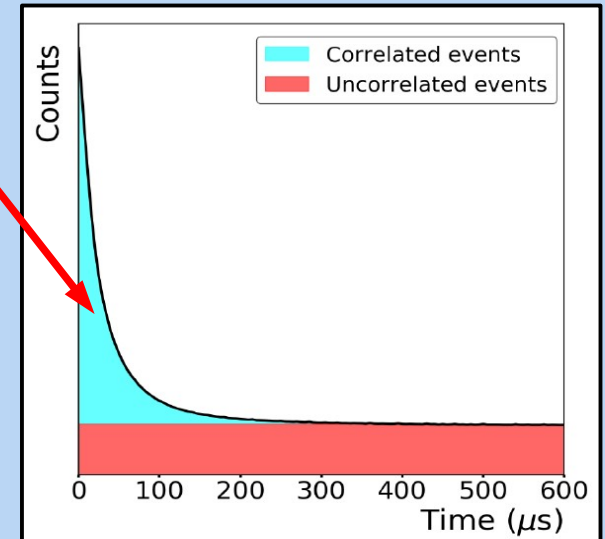
Hall PhD Thesis (2020)

BRIKEN characterization

- We conducted several **calibration measurements** using a ^{252}Cf source.
- **Two** different detector **configurations** (with and w/o HPGe clovers)
- **Innovative digital** implementation of the **Neutron Multiplicity Counting** methodology to determine the detector's efficiency.



$$\varepsilon = \frac{2\nu_{s1}}{\nu_{s2}} \frac{D}{S}$$

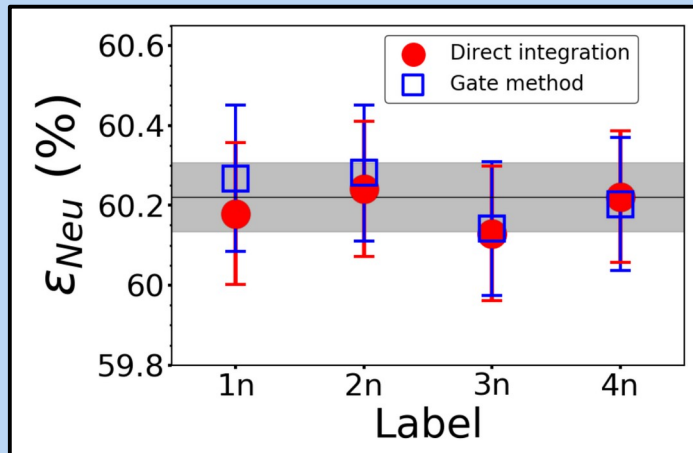


^{252}Cf calibration measurements

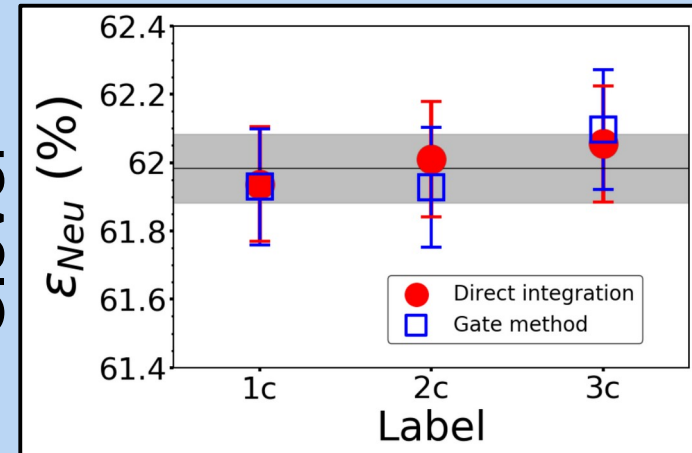
- We employed **two equivalent methods** to evaluate the background that yielded compatible results.
- Sizable effect** of the **clover** presence on the neutron **efficiency**

Setup	ε_{Neu} (%)	σ_{Total} (%)
No Clover	60.19	+0.13
		-0.08
Clover	62.03	+0.12
		-0.06

Non-Clover



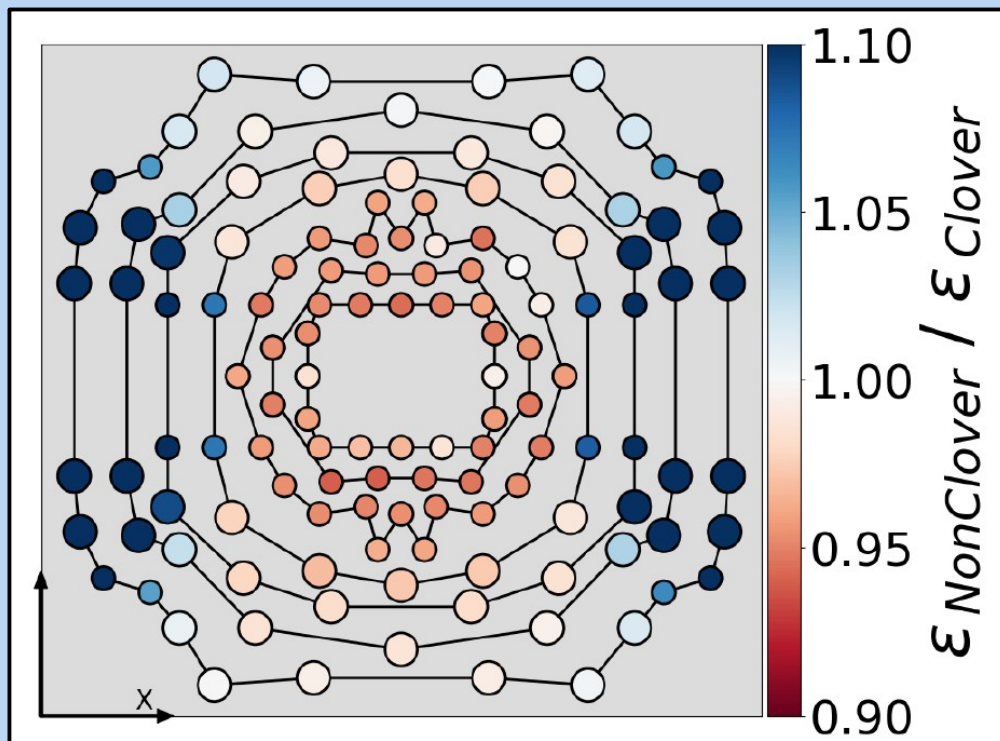
Clover



Neutron transport

The crystal clovers act as a **neutron scatterer**, shadowing the detectors behind them and increasing the efficiency of the inner tubes.

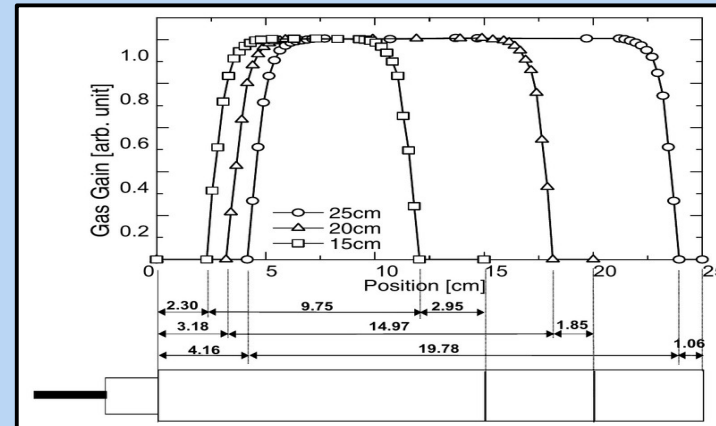
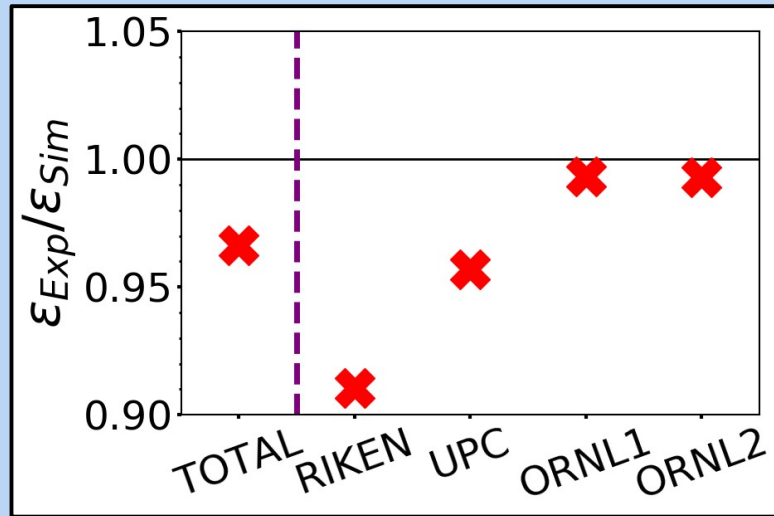
Setup	ε_{Neu} (%)	σ_{Total} (%)
No Clover	60.19	+0.13
		-0.08
Clover	62.03	+0.12
		-0.06



Non clover simulation

	Efficiency (%)	Error(%)
Simulation	62.28	0.03
Experimental	60.21	+0.13 -0.08

- Simulation **overestimation** of $\sim 3.3\%$.
- The discrepancies are produced mainly by the **RIKEN's** and **UPC's** tubes.
- Partially attributed to the **gain reduction** at the edges of the active area of ^3He tubes.



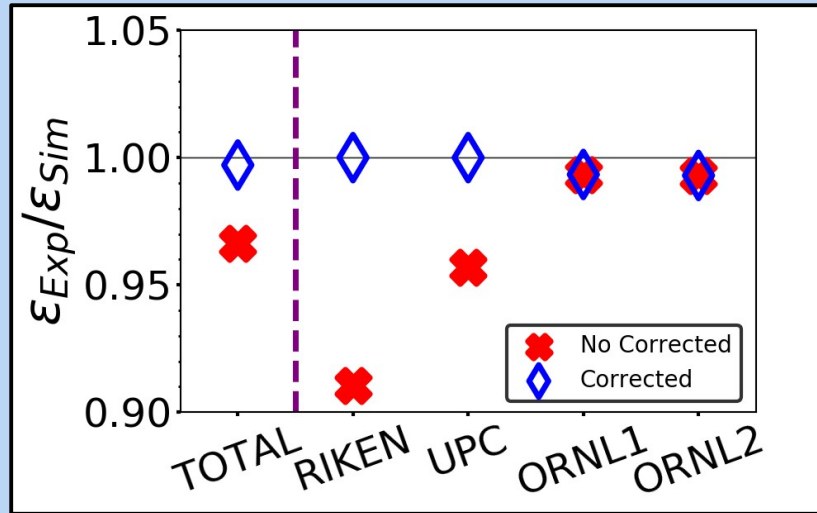
Kudo+ NIMB (2004)

Peplowski+ NIMA (2020)

Non clover simulation

	Efficiency (%)	Error(%)
Simulation	60.38	0.03
Experimental	60.21	+0.13 -0.08

- Simulation **overestimation** of $\sim 3.3\%$.
- The discrepancies are produced mainly by the **RIKEN's** and **UPC's** tubes.
- Partially attributed to the **gain reduction** at the edges of the active area of ^3He tubes.
- Common approach of applying correction factors to **recalibrate** the simulation:
 - 0.91 for the RIKEN tubes.
 - 0.96 for the UPC tubes.

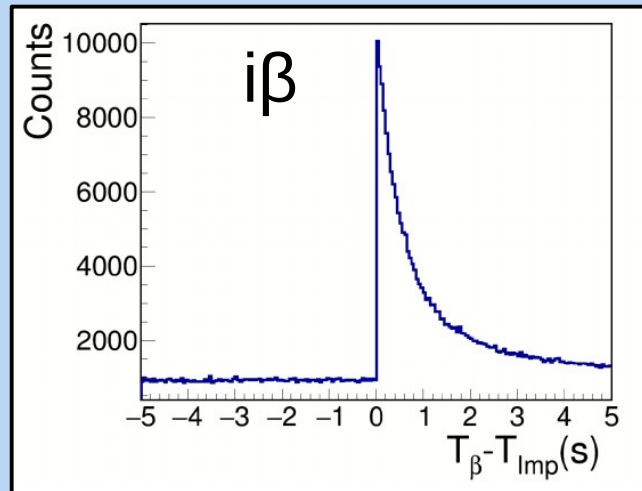


Merger and coincidence curves

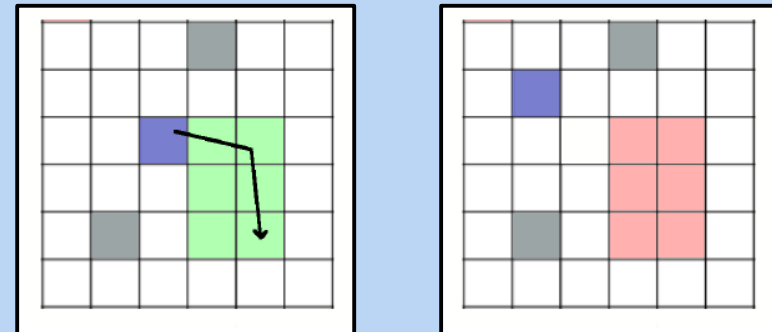
- implant- β coincidence ---> **Decay curve ($i\beta$)**

The analysis is based on the **statistical coincidence** of multiple events.

Tolosa+ NIMA (2019)



Spatial constraints are set to reduce the accidental background.

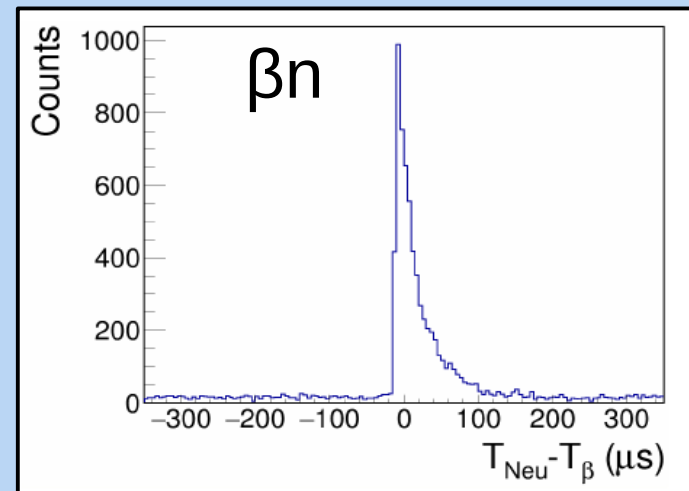
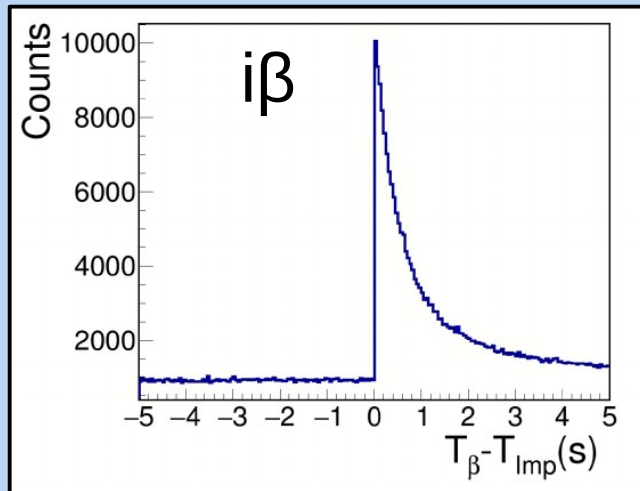


Merger and coincidence curves

- implant- β coincidence ---> **Decay curve ($i\beta$)**
- β -neutron coincidence ---> **Moderation curve (βn)**

The analysis is based on the **statistical coincidence** of multiple events.

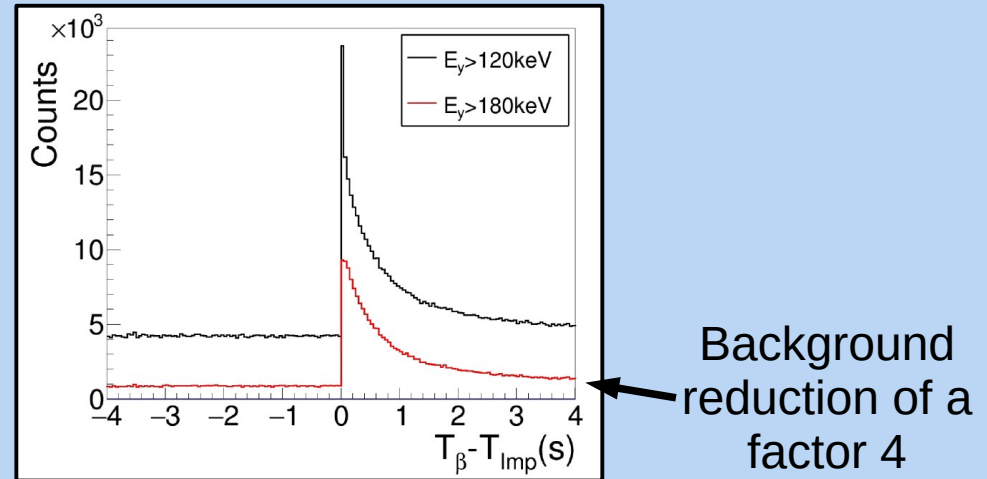
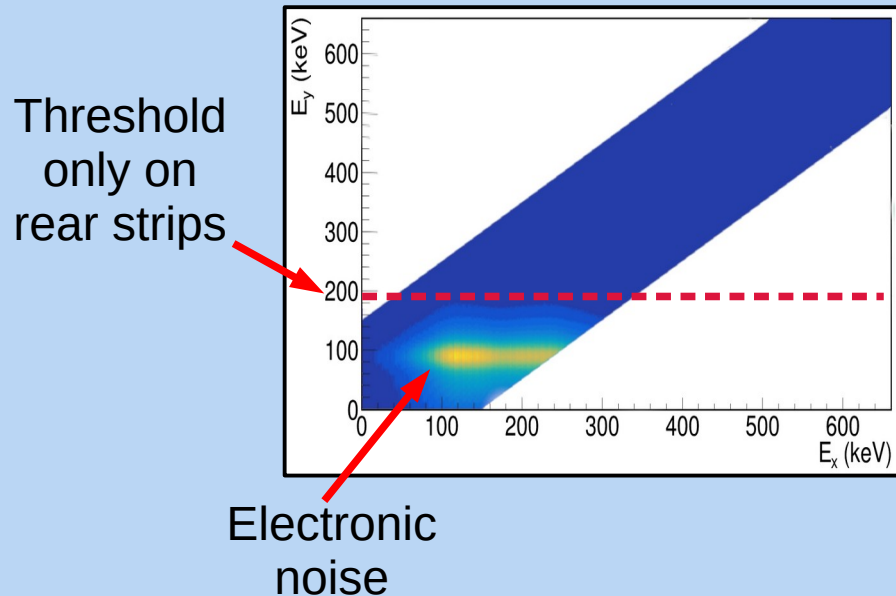
Tolosa+ NIMA (2019)



Improve signal to noise ratio

Methods of **reducing** the background:

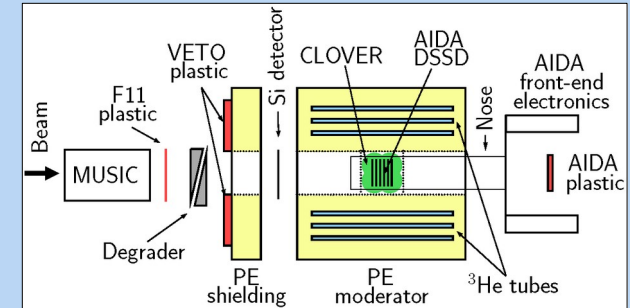
- β energy threshold.



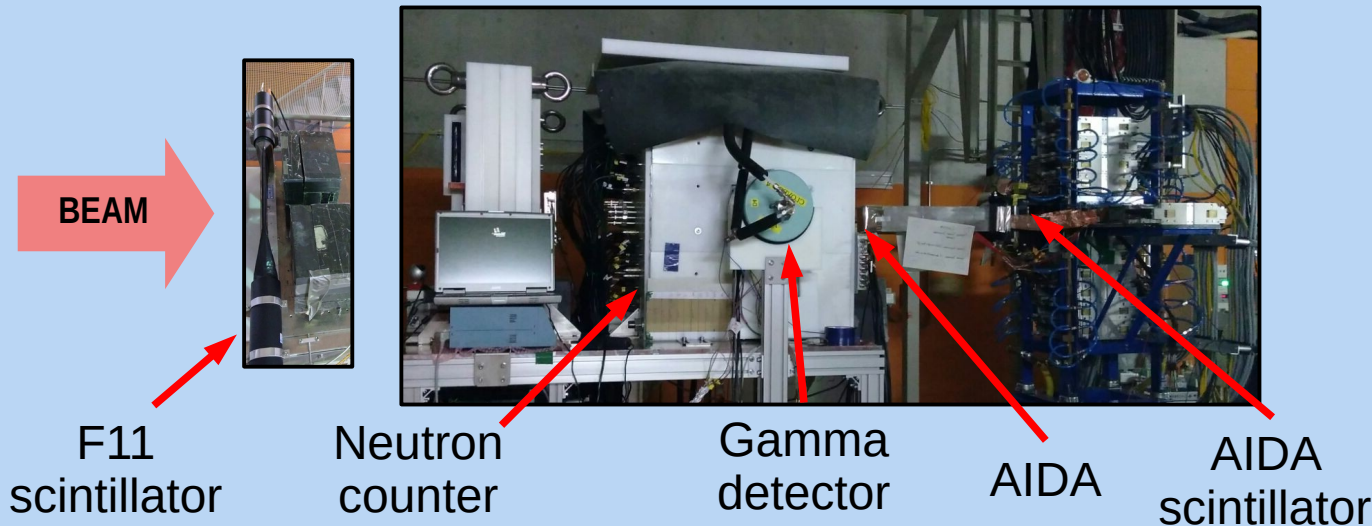
Improve signal to noise ratio

Methods of **reducing** the background:

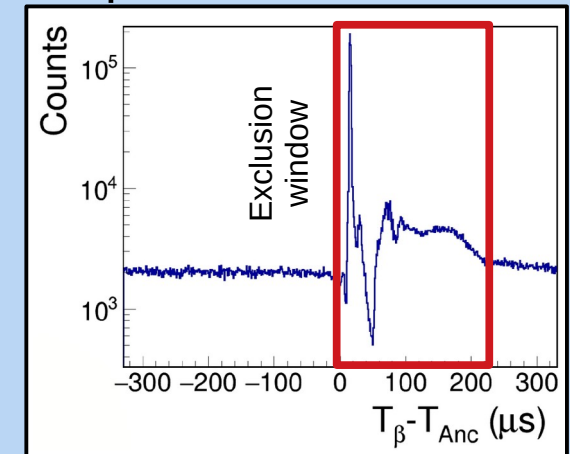
- β energy threshold.
- Build **veto**s for β and neutron events using ancillary detectors.



Tolosa+ NIMA (2019)



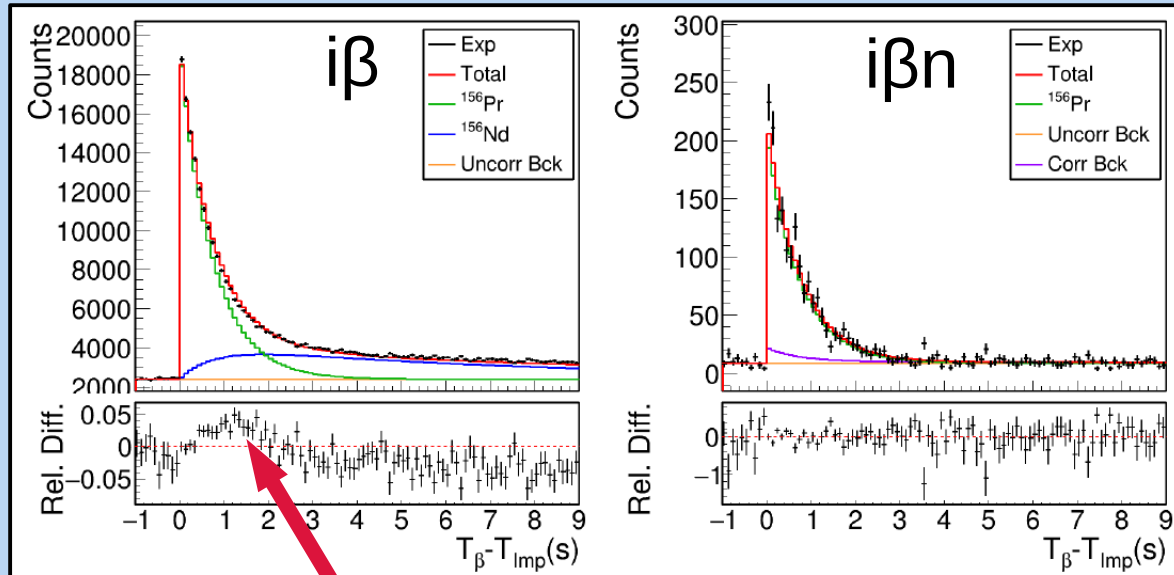
β -anc coincidence



Inconsistent residuum case: ^{156}Pr

- High statistics ($3 \cdot 10^6$ implants)
- Large Q_β (8.8 MeV) and $Q_{\beta n}$ (2.8 MeV)
- Well-measured decay chain

^{156}Pr	Value	σ_{Total}	IAEA
$T_{1/2}(\text{ms})$	511.6	8.1	444 (6)
$P_{1n}(\%)$	1.84	0.12	-



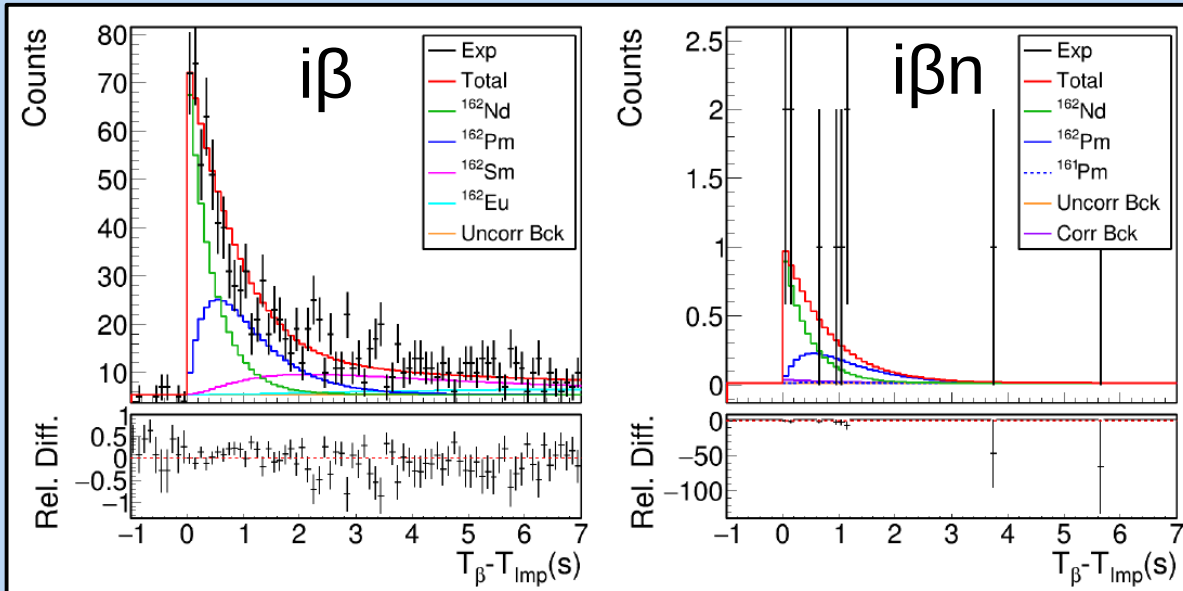
- Clear **systematic deviation** on the residual!

- Large discrepancy with previous measurements.
- Possible existence of an isomeric state.

Low statistic case: ^{162}Nd

- Very low statistics (3000 implants)
- Q_β (7 MeV) and $Q_{\beta n}$ (3 MeV)
- Well-measured decay chain

^{162}Nd	Value	σ_{Total}	IAEA
$T_{1/2}(\text{ms})$	308.1	63.9	310 (200)
$P_{1n}(\%)$	3.73	1.46	-

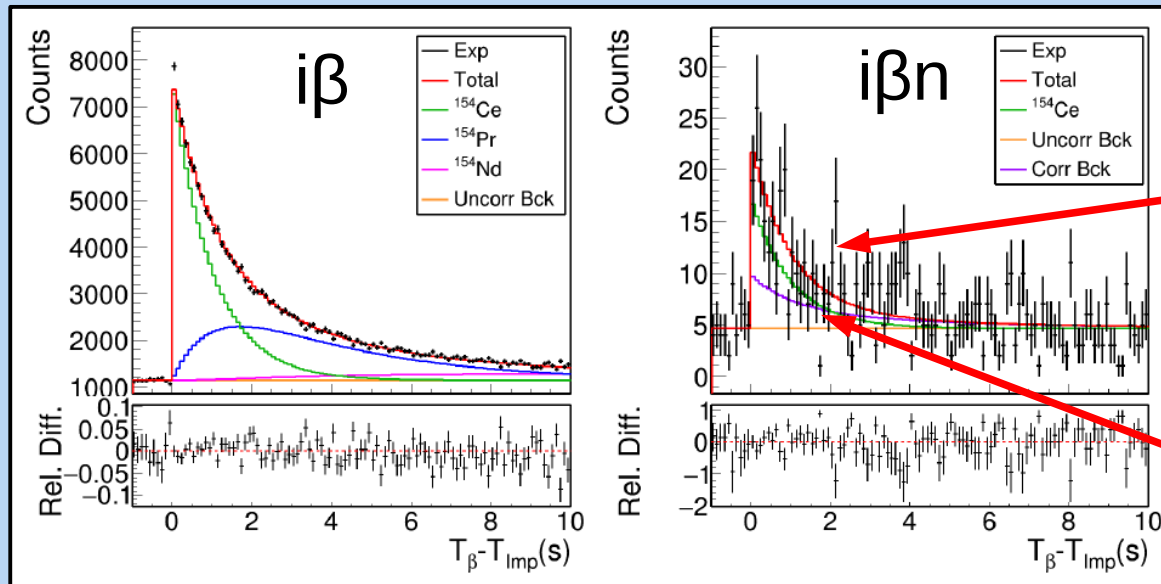


- Result with a large uncertainty but compatible with previous measurements.
- Unreliable 1-neutron gated fit. P_{1n} determined using an alternative method used only for ^{153}Ba and ^{162}Nd .

Small P_{1n} case: ^{154}Ce

- High statistics ($2.4 \cdot 10^5$ implants)
- Small Q_β (5.64 MeV) and $Q_{\beta n}$ (1.28 MeV)
- Decay chain measured with high precision.

^{154}Ce	Value	σ_{Total}	IAEA
$T_{1/2}(\text{ms})$	691.3	8.4	722 (14)
$P_{1n}(\%)$	0.31	0.07	-



Very small P_{1n} but high accuracy

Clear decay curve

Large correlated background

Neutron efficiency

Neutron detection efficiency depends on the **neutron energy spectra**, which is **unknown for all the cases** studied in this thesis.

Assumption that:

$$\langle \varepsilon_n(E) \rangle \approx \varepsilon_n(\langle E \rangle)$$

This thesis nuclei $\langle E \rangle \leq 1$ MeV



Neutron efficiency --> **isotope-independent.**

Nominal eff: 66.5(6) %

Corrections:

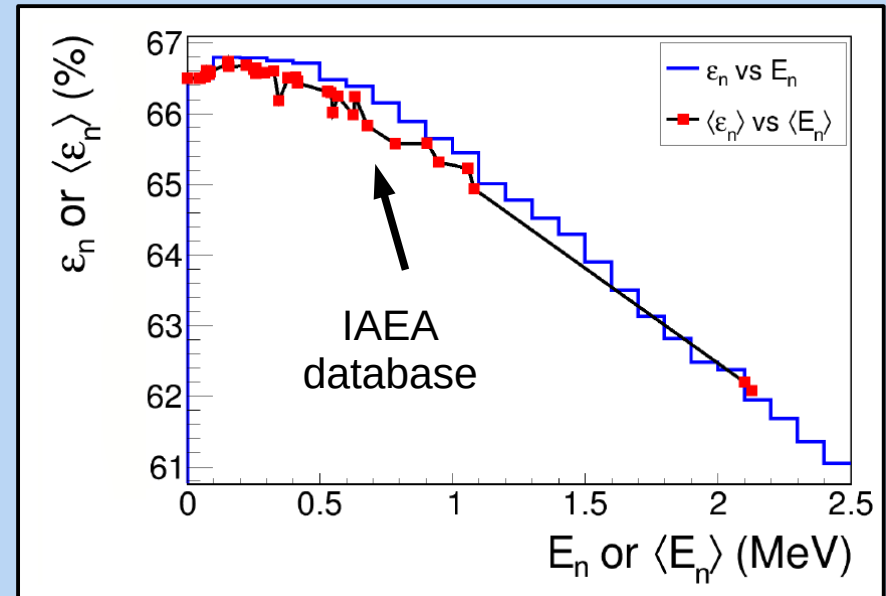
β -neutron coincidence: 98.5%

Veto induced dead time: 95.6%



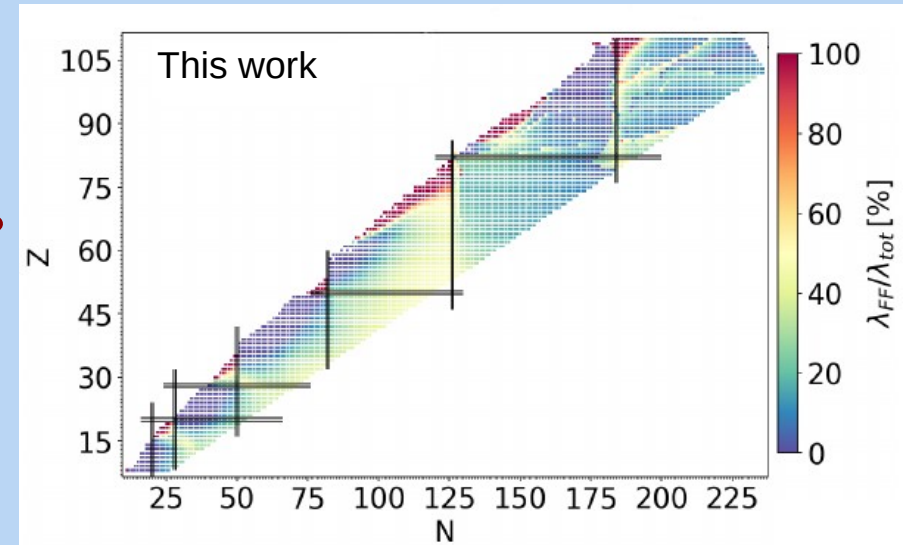
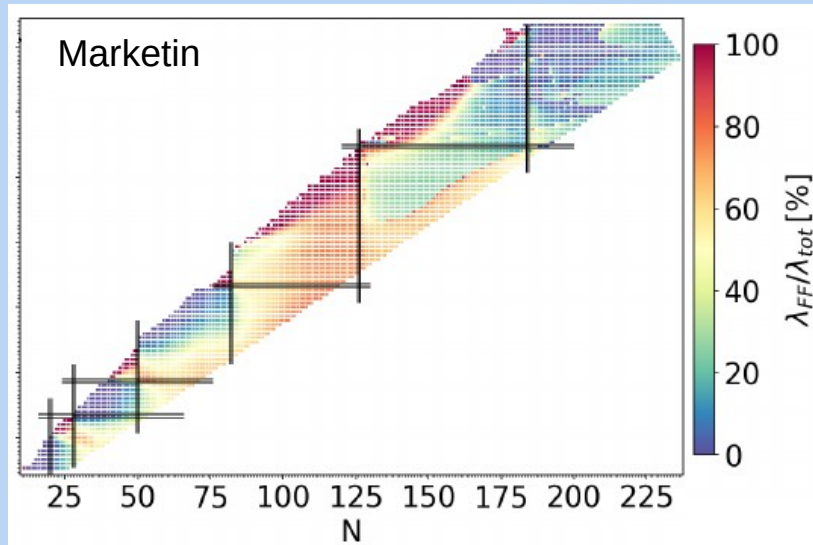
Final effective neutron
efficiency:

$$\varepsilon_n = 62.5(6) \%$$



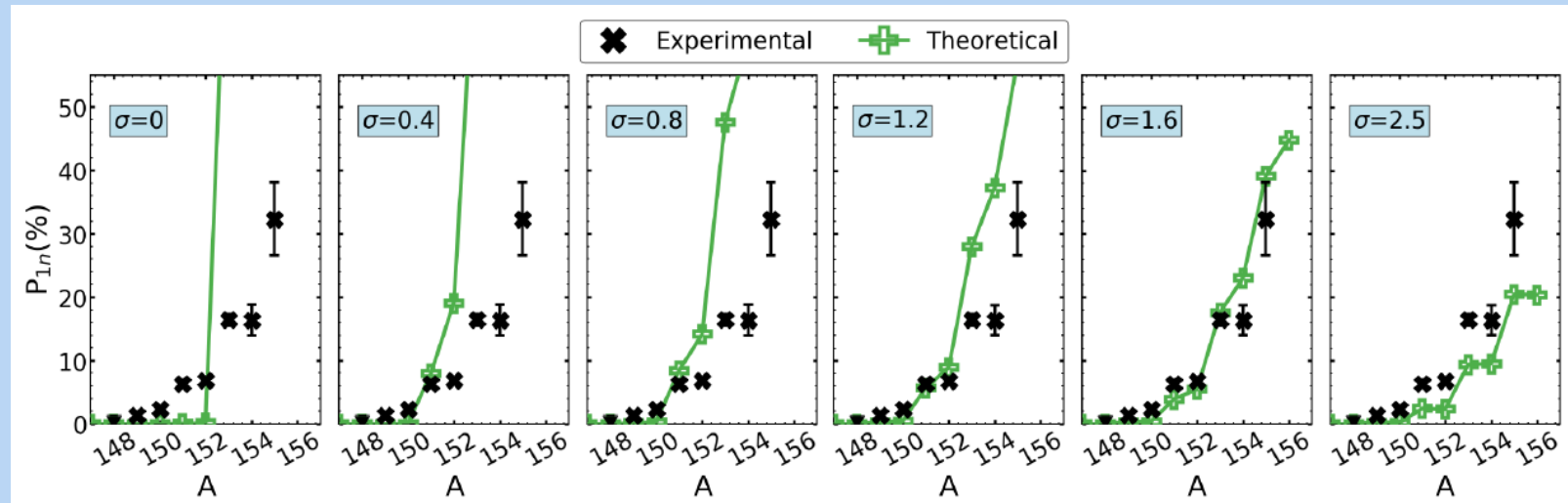
Nuclear structure calculations

- **Revision** of the Marketin et al. Model (QRPA).
- The calculation includes **GT and FF** transitions.



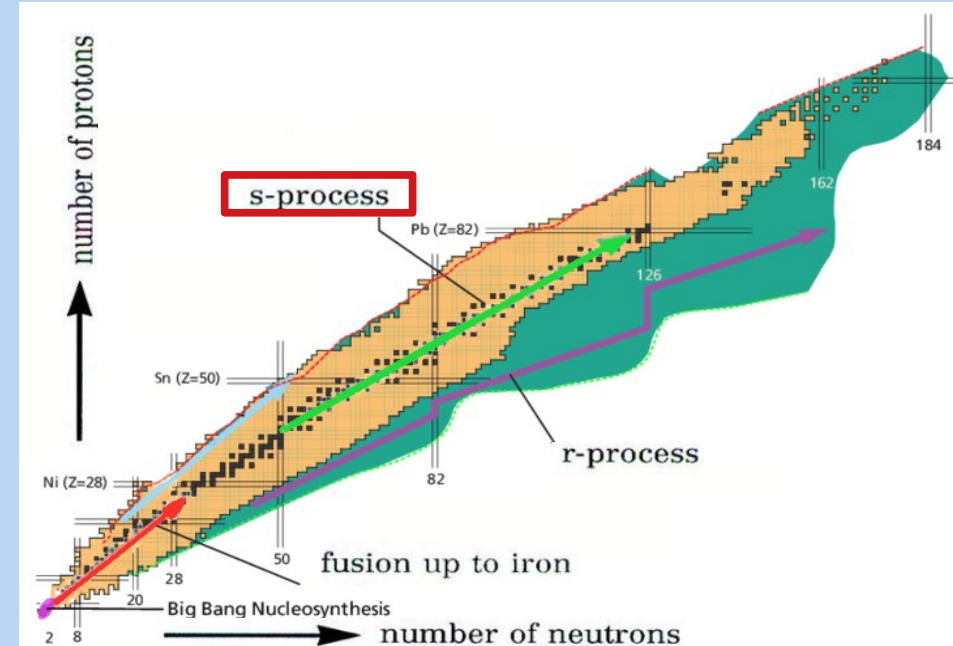
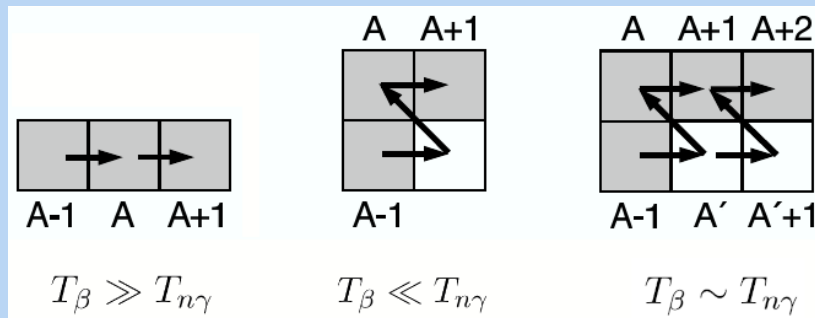
Smearing function

- The QRPA calculation only computes **some** of the excited levels.
- To account for **higher-order terms**, we add a **smearing function** to obtain a smoother distribution.
- The σ is a **free parameter** that can be adjusted to reproduce the experimental data. We selected a value of **1.6 MeV**.



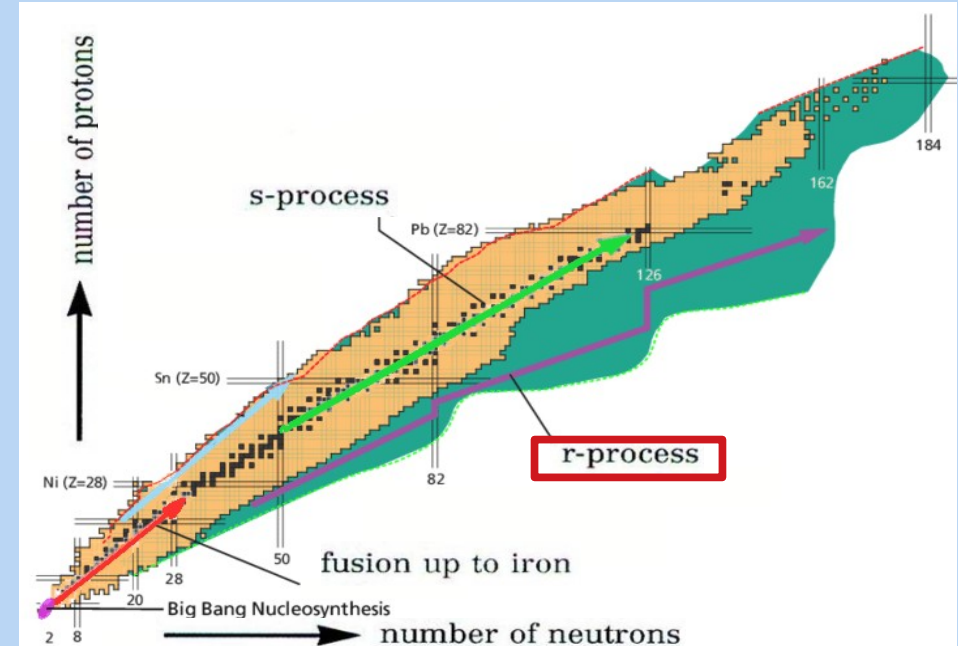
Slow neutron-capture process

- $T_{n\gamma} \gg T_{\beta}$
- Low neutron density.
- Operates along the valley of stability, nuclei nuclei are stable or very long-lived.
- Starts around the iron group.
- This process is extremely slow, operating over hundreds to thousands of years.



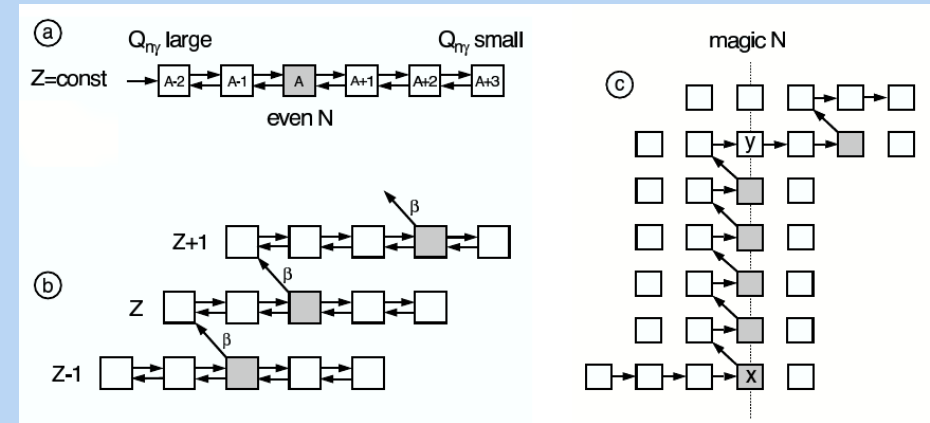
Rapid neutron-capture process

- Extremely high neutron density environment.
- Operates far from the valley of stability, creating very neutron-rich, unstable isotopes.
- This process is fast, operating over μs to ms .



Rapid neutron-capture process

- Extremely high neutron density environment.
- Operates far from the valley of stability, creating very neutron-rich, unstable isotopes.
- This process is fast, operating over μs to ms .
- Three reaction channels compete:
 - Neutron capture (n,γ)
 - Photodissociation (γ,n)
 - Beta-decay



REP importance

On the r-process we can distinguish between two astrophysical environments.

Hot r-process:

High temperatures ($T \sim 1$ GK).

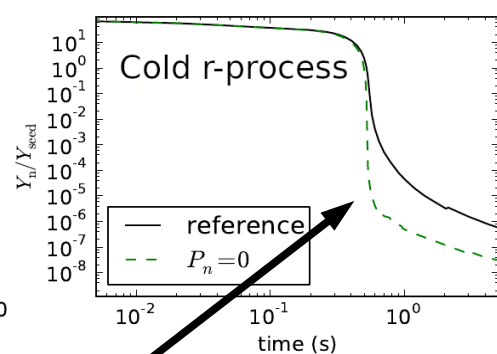
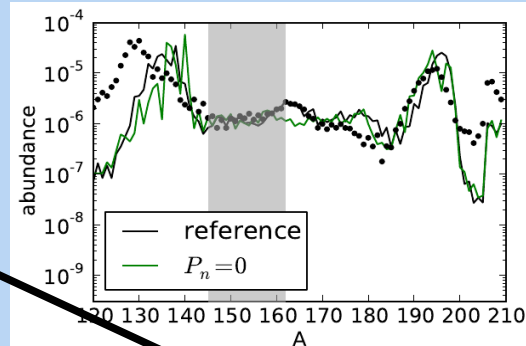
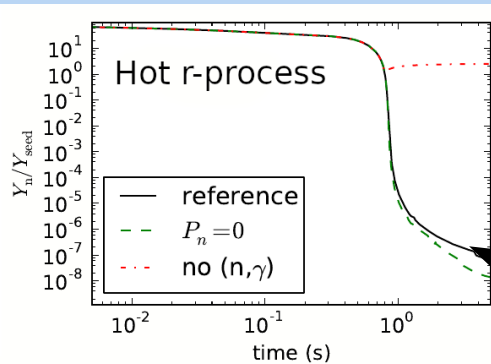
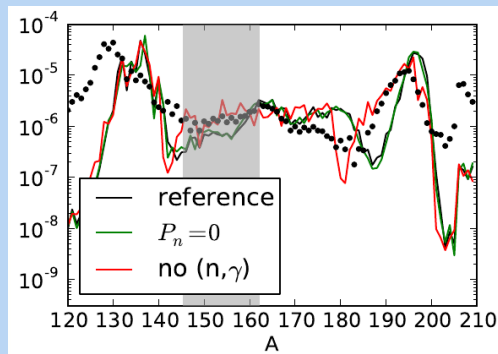
$T_{ny} = T_{yn} \ll T\beta$.

$(n,\gamma) \leftrightarrow (\gamma,n)$ equilibrium.

Cold r-process:

Lower temperatures (< 0.5 GK).

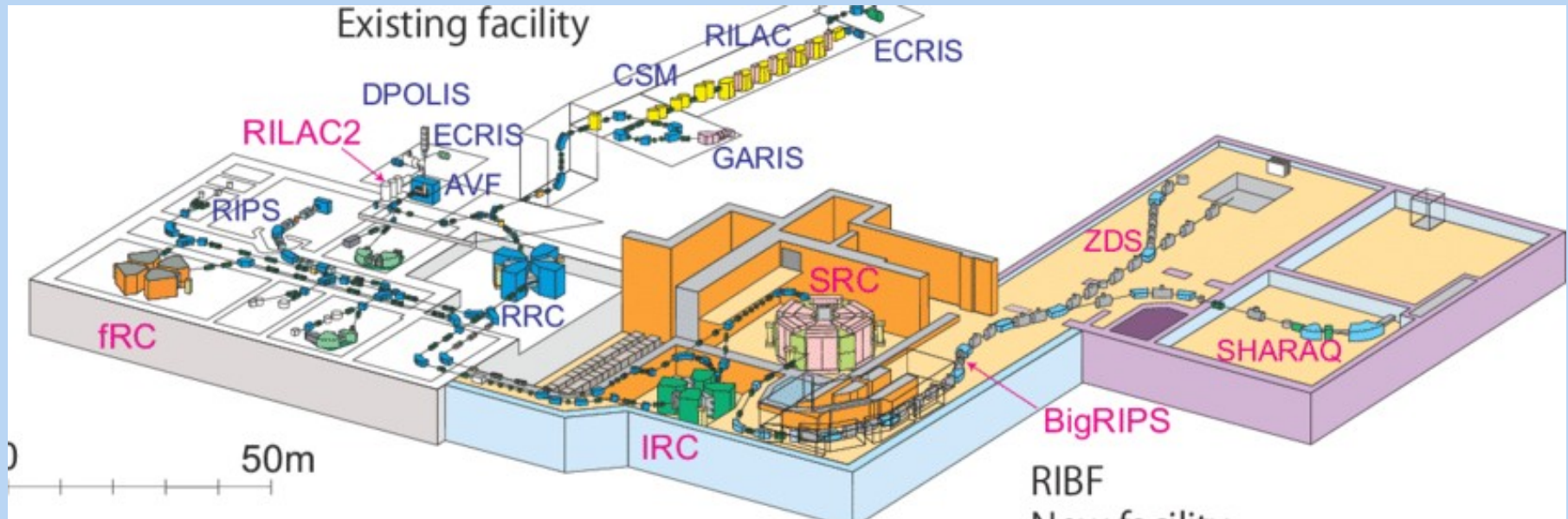
$T_{ny} = T\beta \ll T_{yn}$.



The REP is the only r-process peak formed after the freeze-out phase.

RIKEN Nishina Center

Rare Isotope Beam Factory (RIBF)



BigRIPS

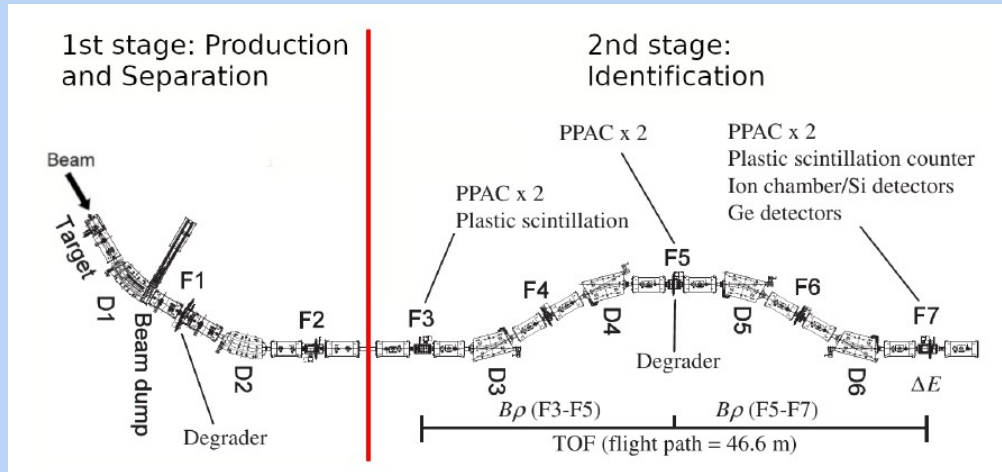
Superconducting Radioactive Isotope Beam Separator

Goal:

- Production
- Separation
- Identification

Setup:

- 14 Superconducting Tripled Quadrupoles
- 6 dipoles magnets
 - Forming 7 focal points
- Position-Sensitive Plate Avalanche Counters (PPACs)
- Multi-Sampling Ionization Chamber (MUSIC)



BigRIPS

The production and separation take place in the first part of BigRIPS (F1 to F3).

Production:

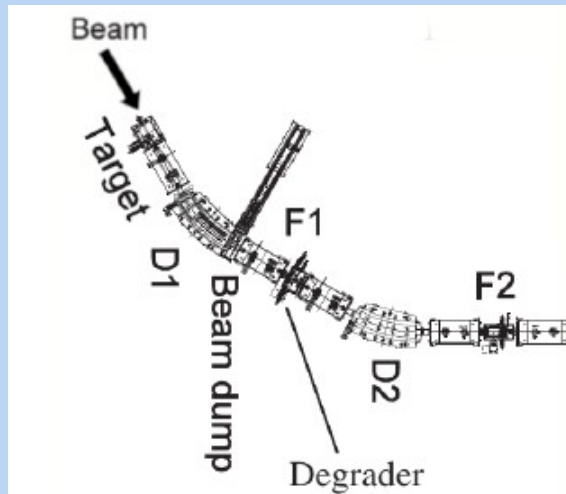
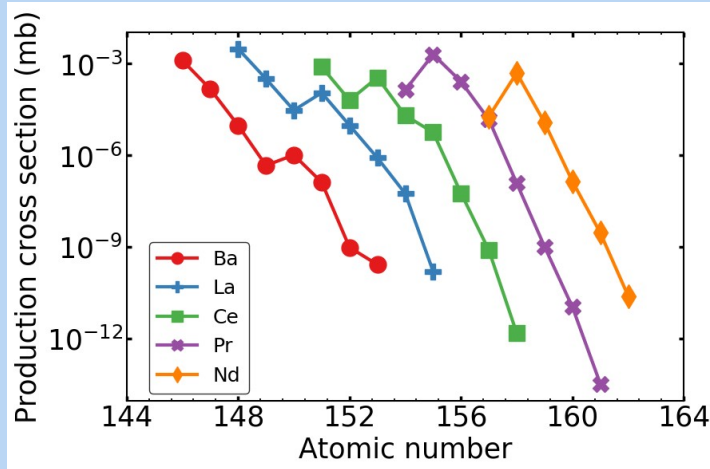
Primary ^{238}U beam: 350MeV/a and 40pnA.

Beryllium target to produce in-flight fission.

Energy-based separation:

Employs the magnetic rigidity of D1 and D2.

The A/Q produces a substantial background removal.



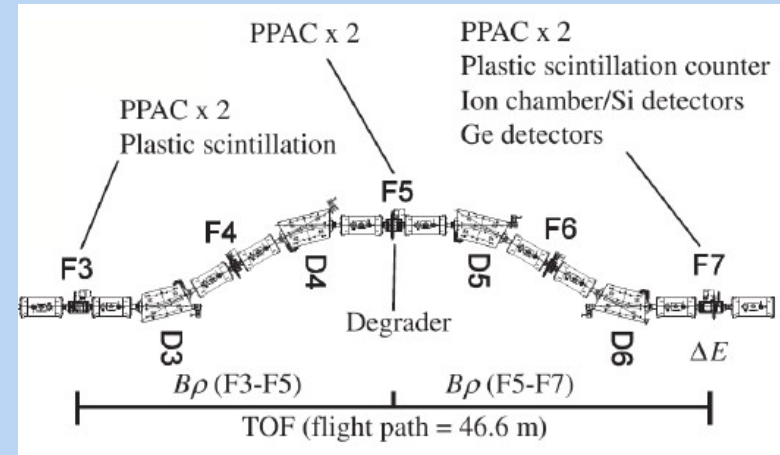
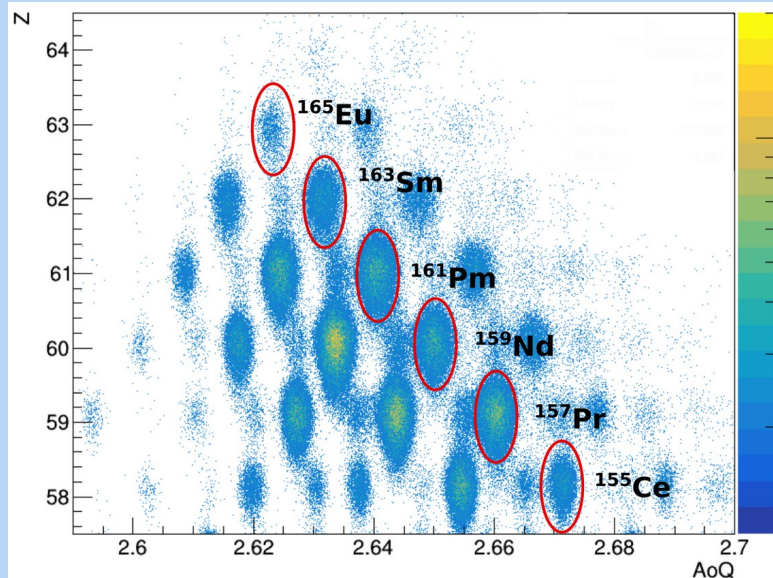
Identification

ToF-B ρ - ΔE method to measure A/Q and Z :

- Plastic scintillators at F3 and F7 --> ToF/velocity
- Magnetic dipoles --> $B\rho$
- MUSIC at F7 --> ΔE

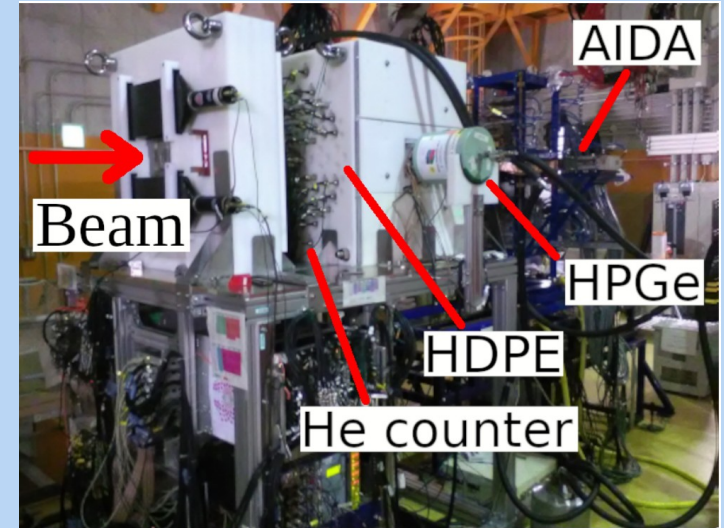
$$\frac{A}{Q} = \frac{B\rho}{\beta\gamma} \frac{c}{m_u}.$$

$$-\frac{dE}{dx} = \frac{4\pi e^4 Z^2}{m_e \beta^2 c^2} n z \left[\ln \left(\frac{2m_e \beta^2 c^2}{I} \right) - \ln(1 - \beta) - \beta^2 \right]$$



Neutron detection: BRIKEN

- 140 ^3He detectors distributed in 7 rings.
- HDPE moderator 90x90x75 cm.
- 2 Clover type HPGe gamma detectors.
- Ancillary (F11, Si...).

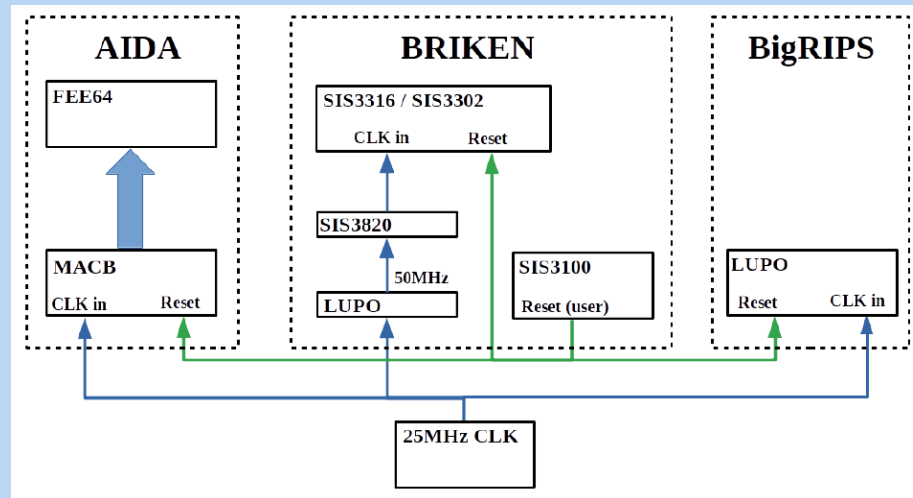


- Acquisition is done using STRUCK modules SIS3316 and SIS3302.
- The modules are controlled by a software called GASIFIC 7.0.

DAQ synchronisation

3 DAQ runs in parallel:

- BigRIPS uses LUPUO.
- BRIKEN uses SIS3316 / SIS3302.
- AIDA uses FEE64.
- Global clock that synchronise the 3 DAQs



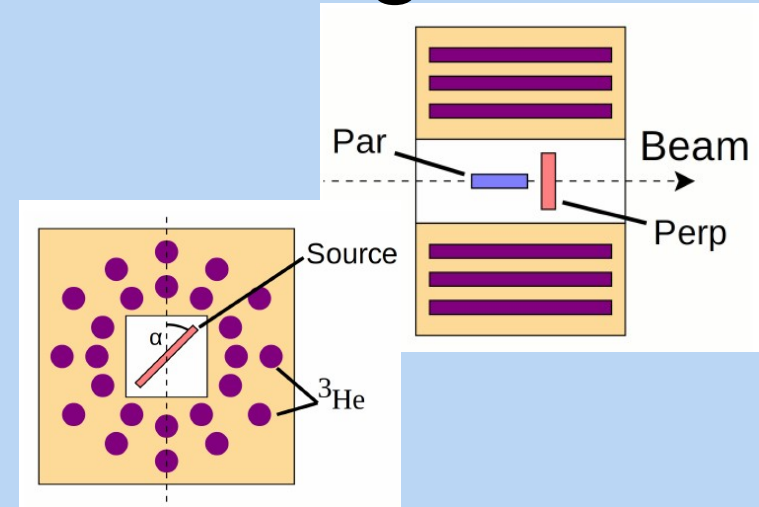
Experimental runs

The RIBF-148 experimental measurements spanned 100 hours. We employed three configurations with three different degrader widths.

Degrader width (mm)	2.8		3		3.3	
Measurement time (h)	64		14		4	
	Implants	r_1	Implants	r_1	Implants	r_1
151Ba	$5.7 \cdot 10^3$	0.0006	$1.8 \cdot 10^3$	-	$2.1 \cdot 10^2$	-
153La	$4.3 \cdot 10^4$	0.0008	$1.3 \cdot 10^4$	0.0007	$1.8 \cdot 10^3$	0.0009
154La	$5.8 \cdot 10^3$	0.0008	$1.7 \cdot 10^3$	0.0008	$2.4 \cdot 10^2$	0.0022
155Ce	$8.3 \cdot 10^4$	0.0008	$2.3 \cdot 10^4$	0.0007	$3.6 \cdot 10^3$	0.0013
156Ce	$2.4 \cdot 10^4$	0.0008	$6.5 \cdot 10^3$	0.0007	$9.6 \cdot 10^2$	0.0011
157Pr	$2.9 \cdot 10^5$	0.0008	$7.8 \cdot 10^4$	0.0007	$1.3 \cdot 10^4$	0.0010
158Pr	$3.3 \cdot 10^4$	0.0007	$9.0 \cdot 10^3$	0.0009	$1.5 \cdot 10^3$	0.0003
159Nd	$2.0 \cdot 10^5$	0.0008	$5.3 \cdot 10^4$	0.0008	$8.8 \cdot 10^3$	0.0011
160Nd	$6.5 \cdot 10^4$	0.0009	$1.7 \cdot 10^4$	0.0008	$2.9 \cdot 10^3$	0.0010

Neutron Multiplicity Counting

- From 2016 to 2018, we conducted several calibration measurements to evaluate the neutron detection.
 - Several angles
 - Several orientation
 - 2 different detectors configuration
- We only had an uncalibrated source.
- We employed the Neutron Multiplicity Counting to determine the detector efficiency.



$$\varepsilon = \frac{2\nu_{s1}}{\nu_{s2}} \frac{D}{S}$$

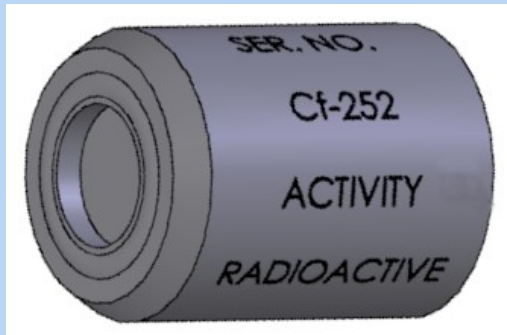
^{252}Cf neutron source

The facility can not facilitate a calibrated source

Impossibility of using an external source

Impossibility of moving the setup to a calibration facility

} We used an
uncalibrated
source



The information given by the manufacturer:

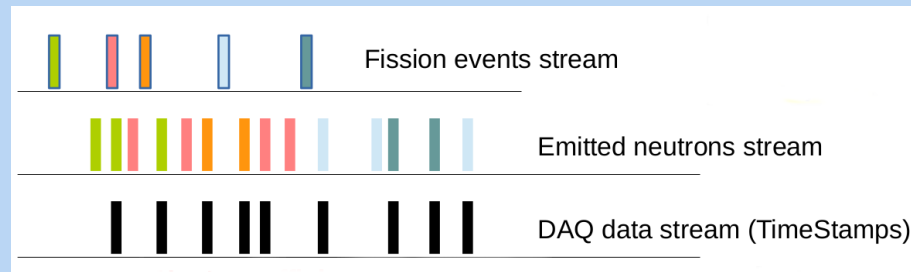
- Nominal activity: 3.7 Mbq (with large undetermined uncertainty)
- Impurities: ^{249}Cf =0.0631%, ^{250}Cf =7.69% and ^{251}Cf =0.037%

Neutron Multiplicity Counting

This method advantages:

- Does not require of a calibrated source.
- Offers uncertainties around 1% for the neutrons' yield determination.
- Comparable to metrological standard techniques.
- Digital electronics
- ^{252}Cf fission properties are well characterized.
- BRIKEN detector has high efficiency.

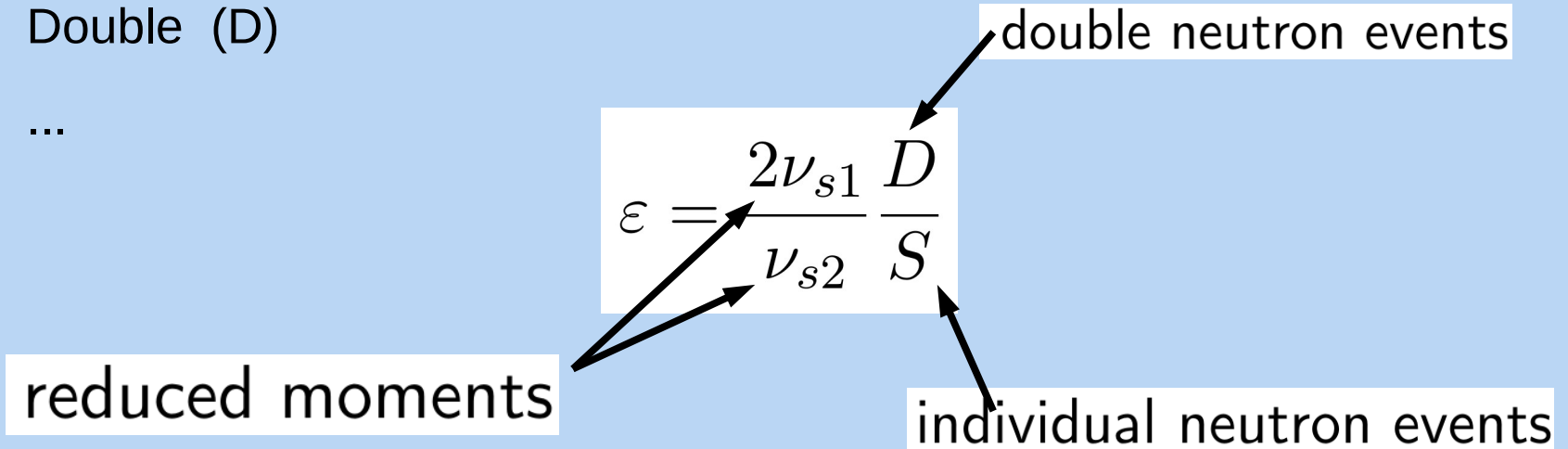
^{252}Cf undergoes spontaneous fission, emitting multiple neutrons time-correlated neutrons.



Neutron Multiplicity Counting

From this individual fission events one can detect groups of correlated neutrons:

- Singles (S)
- Double (D)
- ...



Rossi-alpha distribution

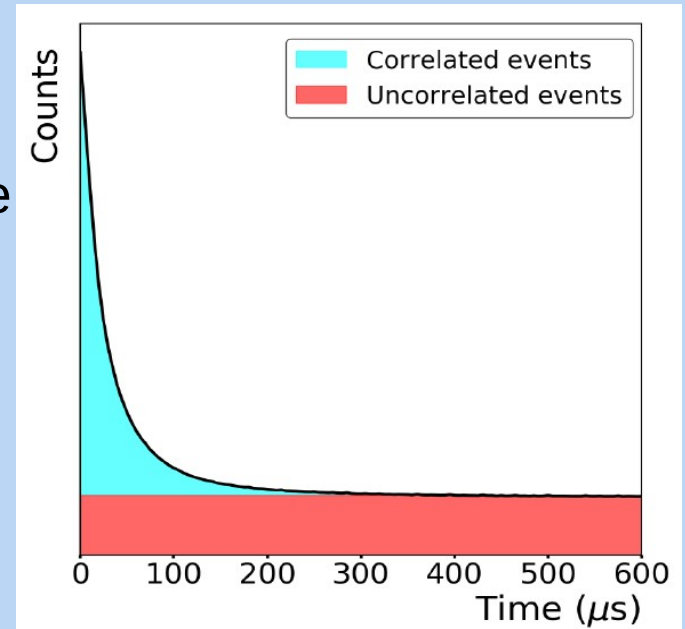
Sort the time differences between each detected neutron and all subsequent neutrons within a defined time window.

Non-correlated neutron coincidence --> Flat background.

Correlated neutron coincidence --> More likely to be detected within short time intervals.

From this plot, we compute D with two equivalent methodologies

- Direct Integration
- Gate method



Results error

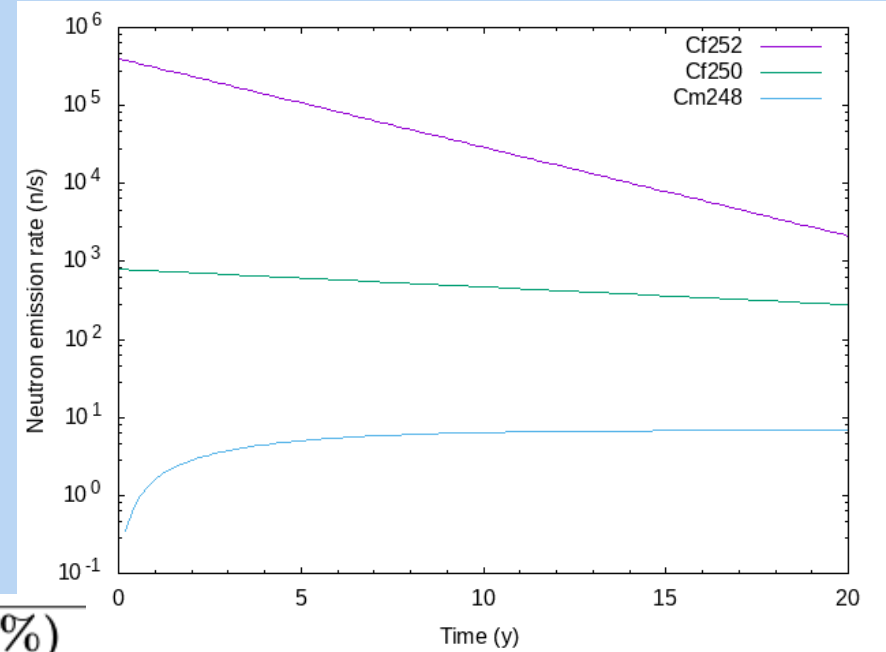
Systematic errors:

- Source contamination: Contribution of ^{250}Cf included as +0.1 of error.
- Source anisotropy: negligible due to the 4pi detector setup.
- Dead time: negligible due to very low detection rate (<200 cps).

Setup	ε_{Neu} (%)	σ_{Total} (%)	σ_{Stat} (%)	σ_{Syst} (%)
No Clover	60.19	+0.13 -0.08	± 0.08	+0.1
Clover	62.03	+0.12 -0.06	± 0.06	+0.1

Isotopes contamination

- The main contaminants are ^{250}Cf and ^{248}Cm .
- In the period considered only ^{250}Cf has a measurable effect.
- The correction can be rounded to +0.1%



Month/Year	Absolute efficiency correction (%)
------------	------------------------------------

07/2016	+0.07
---------	-------

06/2017	+0.09
---------	-------

11/2018	+0.11
---------	-------

Simulations

MC simulations to characterize our detectors' efficiency.

- Used ParticleCounter 4.7.
- Compiled with Geant4 10.1.3.
- G4DL4.5 data library.

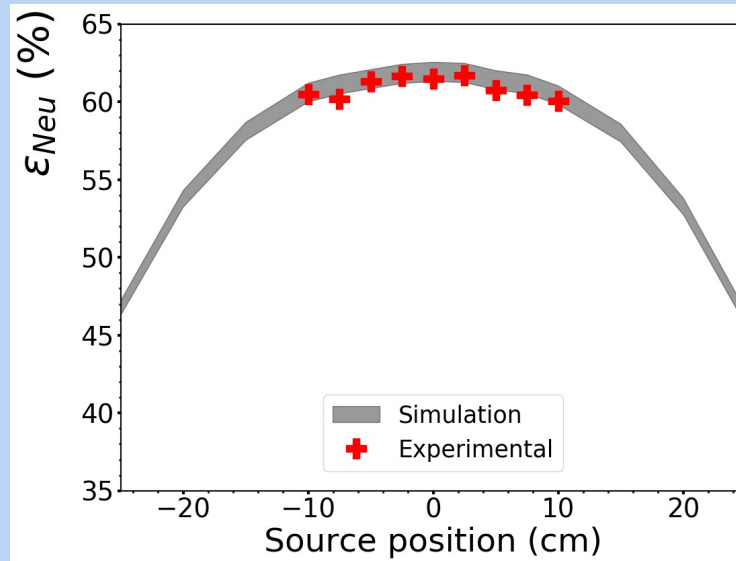
We simulated both geometries (non-clover/clover) with a detailed description of the ^3He detectors and a simplified version of the HPGe germanium crystals.

^{252}Cf spectrum recommendation from Manhart

- Benchmark with the experimental calibrations

BRIKEN energy response

- No effect on Z (missplacement of AIDA).



AIDA sorting

AIDA consist
of:

6 **DSSDs**



Each with two
sides of **128 strips**



Each is registered by a
self-trigger channel



No interchannel event
correlation. Only a list
of time-ordered items.



Correlation is build via
software: **AIDAsort**



AIDA sorting: cluster information

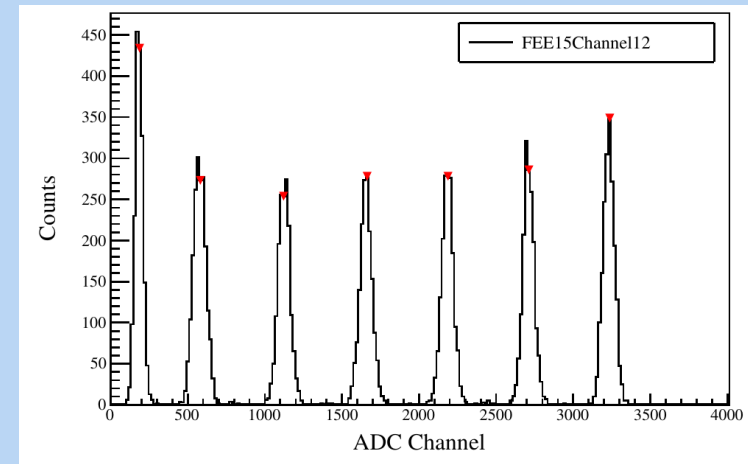
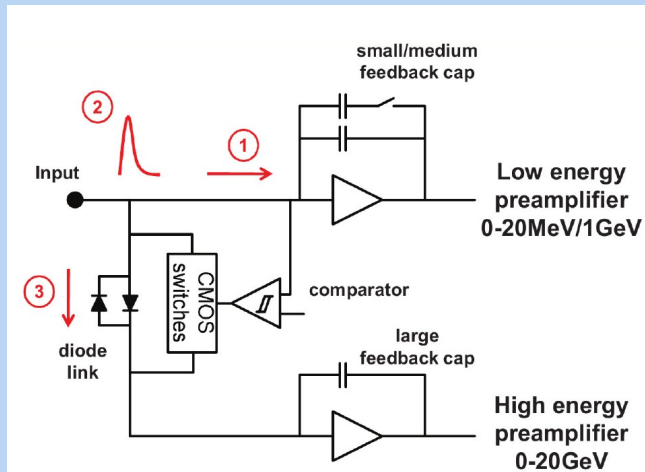
Event type:

≥ 1 high-energy item:
Implantation event

No high-energy item: Low-
energy event

Stopping layer:

Defined as the last stack that
has triggered the high-energy
branch.



AIDA sorting: cluster information

Event type:

≥ 1 high-energy item:

Implantation event

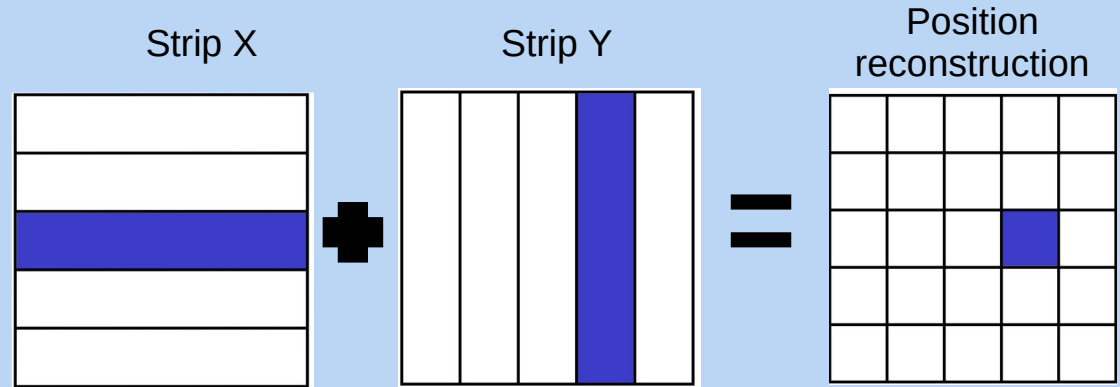
No high-energy item: Low-energy event

Position:

Using the double side detectors clusters have a trigger in x and y, therefore one can define a “pixel”.

Stopping layer:

Defined as the last stack that has triggered the high-energy branch.



AIDA sorting: cluster information

Event type:

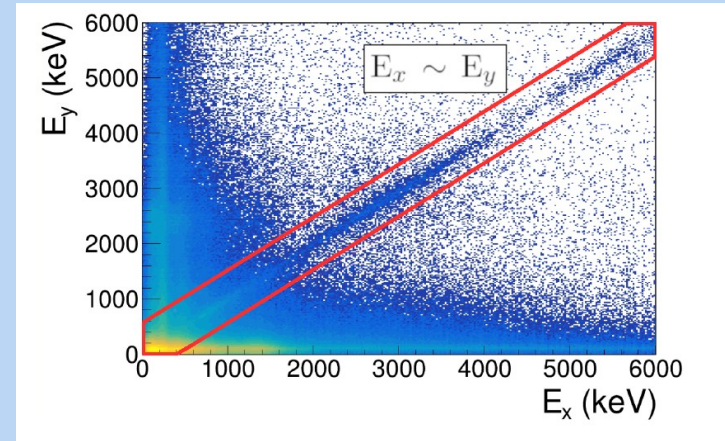
≥ 1 high-energy item:

Implantation event

No high-energy item: Low-energy event

Position:

Using the double side detectors clusters have a trigger in x and y, therefore one can define a “pixel”.



Energy:

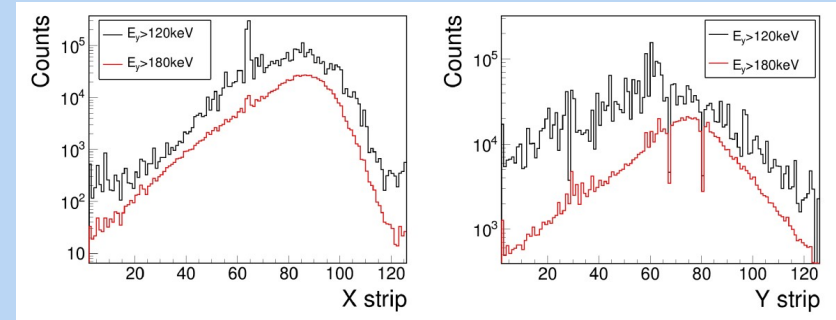
Addition of all the items' energy included in the cluster.

A consistency condition is imposed
 $E_x \sim E_y$

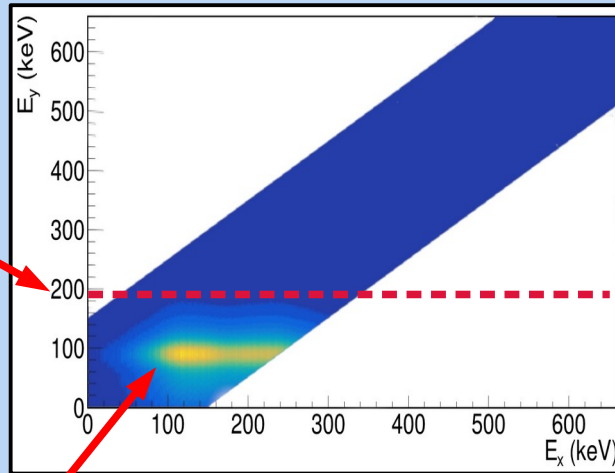
Improve signal to noise ratio

Methods of **reducing** the background:

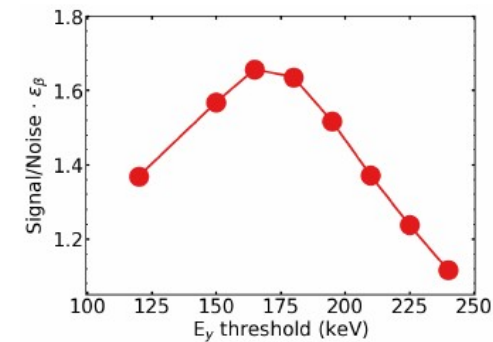
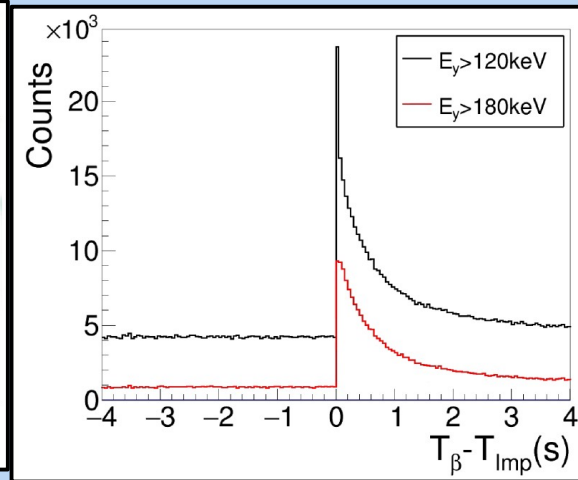
- β energy threshold.



Threshold
only on
rear strips



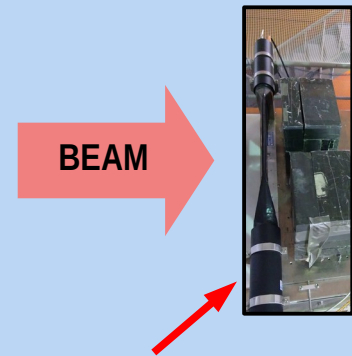
Electronic
noise



Improve signal to noise ratio

Methods of **reducing** the background:

- β energy threshold.
- Build **veto**s for β and neutron events using ancillary detectors.

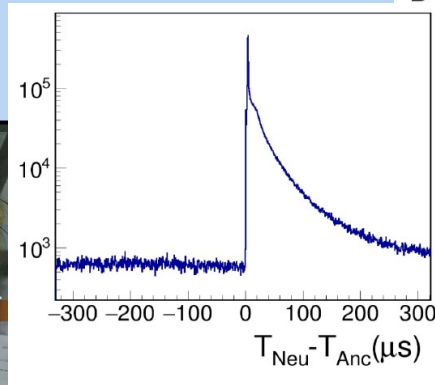


F11
scintillator



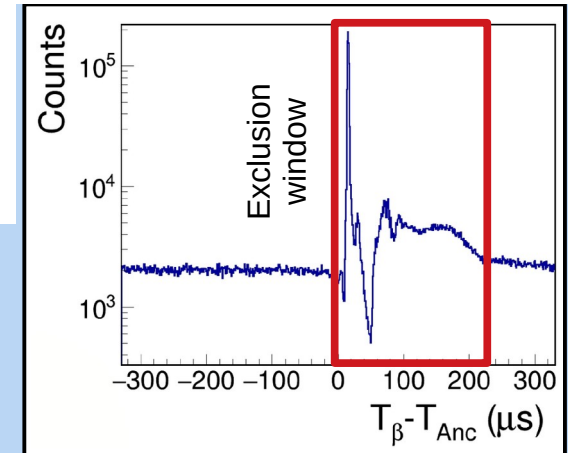
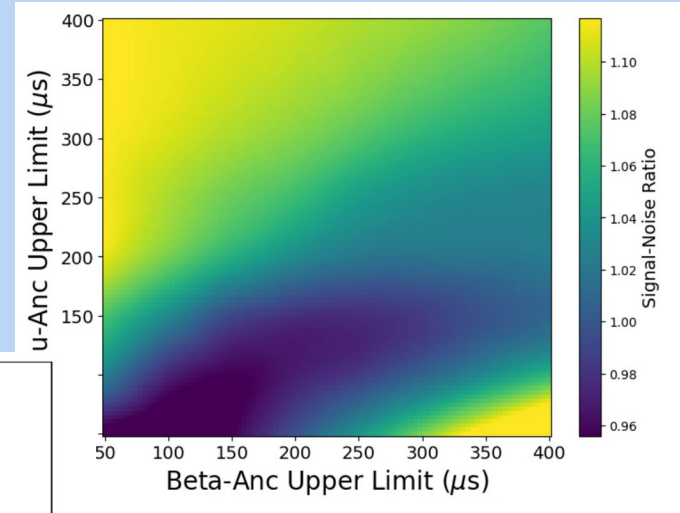
Neutron
counter

Gamma
detector



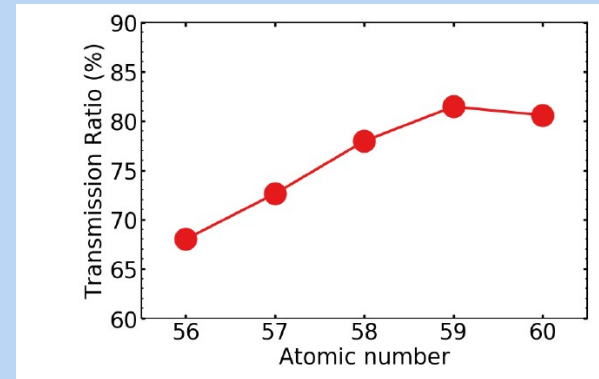
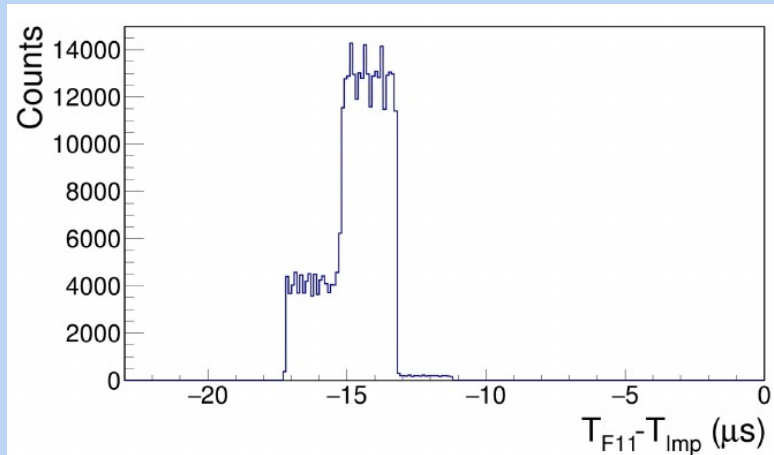
AIDA

AIDA
scintillator

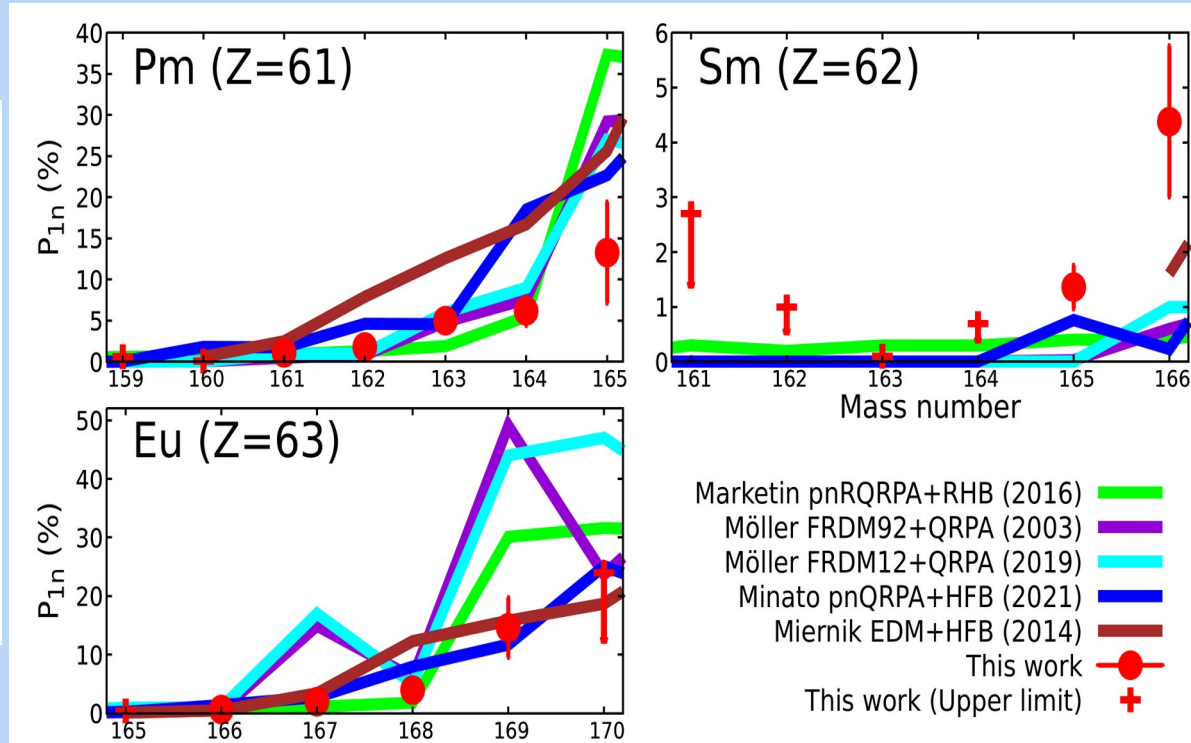
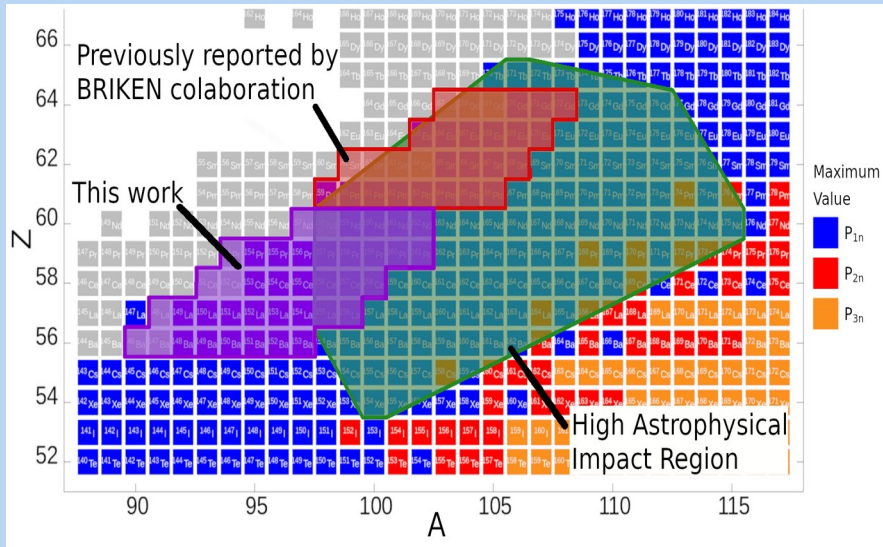


Merger and correlation curves

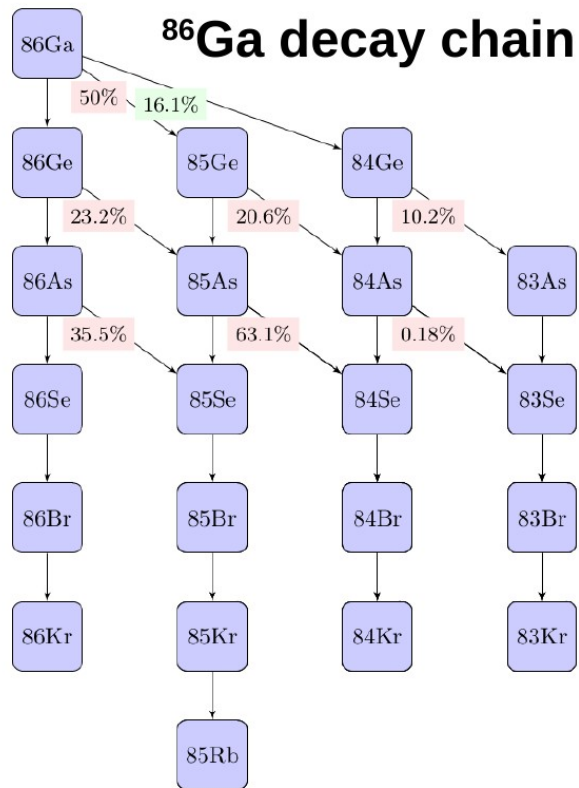
- Transferring the identification in BigRIPS to AIDA.
- We can assume instantaneous transmission from F11 to AIDA.
- Some ions do not reach AIDA due to the beam dispersion.



Other P_{1n} results from BRIKEN



G. G. Kiss, et al., ApJ936 (2022)



Fit function: Example from A. Tolosa

$$f_{\beta}(t) = \sum_{i \in \beta} \bar{\varepsilon}_{\beta}^i \lambda_i N_i(t) + \text{bck}_{\beta}(t)$$

$$f_{\beta 1n}(t) = \sum_{j \in \beta 1n} \bar{\varepsilon}_{\beta}^j \bar{\varepsilon}_n^j P_{1n}^j \lambda_j N_j(t) + \text{bck}_{1n}(t)$$

$$f_{\beta 2n}(t) = \sum \bar{\varepsilon}_{\beta}^k \left(\bar{\varepsilon}_n^k \right)^2 P_{2n}^k \lambda_k N_k(t) + \text{bck}_{2n}(t)$$

$$N_k(t) = N_1 \prod_{i=1}^{k-1} (b_{i,i+1} \lambda_i) \times \sum_{i=1}^k \frac{e^{-\lambda_i t}}{\prod_{j=1 \neq i}^k (\lambda_j - \lambda_i)}$$

$$b_{i,i+1} = P_{1n}^i, P_{2n}^i \text{ or } 1 - P_{1n}^i - P_{2n}^i$$

- ε are averaged efficiencies (weighted by β -intensity distribution)
- Only number of decays & parent P_{xn} (& $T_{1/2}^*$) are set free during the fit.
- The rest of parameters ($T_{1/2}$ & P_{xn} of other isotopes, efficiencies, background) are fixed.

Theoretical models

RHB Relativistic Hartree Bogoliubov

FRDM: Finite Range Droplet Model for mass model

QRPA: Quasi random particle approximation

HFB: Hartree-Fock-Bogoliubov

HF: Hauser-Feshbach

pnFAM: proton-neutron Finite Amplitude Method

pnRQRPA: proton-neutron Relativistic Quasiparticle Random-Phase Approximation

EDM: Effective density model.

Nuclear structure calculations

- Revision of the Marketin et al. model:
 - Self-consistent Covariant Density-Functional Theory (CDFT)
 - Ground-states: RHB model with the D3C* parametrization.
 - Excited states: pn-RQRPA

$$k = [\zeta_0^2 + \frac{1}{9}w^2]_{(0)} + [\zeta_1^2 + \frac{1}{9}(x+u)^2 - \frac{4}{9}\mu_1\gamma_1u(x+u) + \frac{1}{18}W_0^2(2x+u)^2 - \frac{1}{18}\lambda_2(2x-u)^2]_{(1)} + [\frac{1}{12}z^2(W_0^2 - \lambda_2)]_{(2)}, \quad (11a)$$

$$ka = [-\frac{4}{3}uY - \frac{1}{9}W_0(4x^2 + 5u^2)]_{(1)} - [\frac{1}{6}W_0z^2]_{(2)}, \quad (11b)$$

$$kb = \frac{2}{3}\mu_1\gamma_1\{-[\zeta_0w]_{(0)} + [\zeta_1(x+u)]_{(1)}\}, \quad (11c)$$

$$kc = \frac{1}{18}[8u^2 + (2x+u)^2 + \lambda_2(2x-u)^2]_{(1)} + \frac{1}{12}[(1+\lambda_2)z^2]_{(2)}, \quad (11d)$$

In the Condon-Shortley phase convention [48] the matrix elements for β^- transitions are

$$w = -g_A\sqrt{3}\frac{\langle f || \sum_k r_k [C_1^k \otimes \sigma^k]^0 t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13a)$$

$$x = -\frac{\langle f || \sum_k r_k C_1^k t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13b)$$

$$u = -g_A\sqrt{2}\frac{\langle f || \sum_k r_k [C_1^k \otimes \sigma^k]^1 t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13c)$$

$$z = 2g_A\frac{\langle f || \sum_k r_k [C_1^k \otimes \sigma^k]^2 t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13d)$$

$$w' = -g_A\frac{2}{\sqrt{3}}\frac{\langle f || \sum_k r_k I(1,1,1,1,r_k)[C_1^k \otimes \sigma^k]^0 t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13e)$$

$$x' = -\frac{2}{3}\frac{\langle f || \sum_k r_k I(1,1,1,1,r_k)C_1^k t_-^k || i \rangle}{\sqrt{2J_i + 1}}, \quad (13f)$$

$$u' = -g_A\frac{2\sqrt{2}}{3}\frac{\langle f || \sum_k r_k I(1,1,1,1,r_k)[C_1^k \otimes \sigma^k]^1 t_-^k || i \rangle}{\sqrt{2J_i + 1}}. \quad (13g)$$

Nuclear structure calculations

$$E_{g.s.}^b(Z+1, N-1) \approx E_{g.s.}^b(Z, N) + \left. \frac{dE}{dZ} \right|_{(Z,N)} + \left. \frac{dE}{dN} \right|_{(Z,N)} + E_{2qp}^{\text{lowest}}$$

Ground-State
binding energy

Lowest two-
quasiparticles
energies

By definition

Chemical potential $\rightarrow$

$$\lambda_p = \left. \frac{dE}{dZ} \right|_{(Z,N)} \quad (6.6)$$

$$\lambda_n = \left. \frac{dE}{dN} \right|_{(Z,N)}$$

Therefore

$$\Delta B = E_{g.s.}^b(Z, N) - E_{g.s.}^b(Z+1, N-1) \approx \lambda_n - \lambda_p - E_{2qp}^{\text{lowest}} \quad (6.7)$$

Where λ_p and λ_n represent the proton and neutron Fermi energies, respectively. The correction term E_{2qp}^{lowest} accounts for odd-even effects and is determined as follows based on the parent nucleus composition:

$$E_{2qp}^{\text{lowest}} = \begin{cases} E_{\pi}^{\text{lowest}} + E_{\nu}^{\text{lowest}} & (\text{even-even parent}) \\ E_{\nu}^{\text{lowest}} & (\text{even-odd parent}) \\ E_{\pi}^{\text{lowest}} & (\text{odd-even parent}) \\ 0 & (\text{odd-odd parent}) \end{cases} \quad (6.8)$$

Here, E_{π}^{lowest} (E_{ν}^{lowest}) denotes the unperturbed proton (neutron) quasiparticle energies.

Combining equations 6.4 and 6.7, we can compute the energy conversion from the QRPA frame to the daughter reference as follows:

$$E_m^* \approx E_m^{\text{QRPA}} + \Delta B - (\lambda_n - \lambda_p) = E_m^{\text{QRPA}} + Q_{\beta} - \Delta M_{nH} - (\lambda_n - \lambda_p) \quad (6.9)$$

Nuclear structure calculations

- Revision of the Marketin et al. model:
 - Correction on the phase integral limits (missing m_e)
 - Proper implementation of the residual interactions.
 - Correction of some phases on the matrix elements.

$$\begin{pmatrix} A^J & B^J \\ B^{*J} & A^{*J} \end{pmatrix} \begin{pmatrix} X^{\lambda J} \\ Y^{\lambda J} \end{pmatrix} = E_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^{\lambda J} \\ Y^{\lambda J} \end{pmatrix}. \quad (3.28)$$

The matrices A and B are defined in the canonical basis [9]

$$\begin{aligned} A_{pn,p'n'}^J &= H_{pp'}^{11} \delta_{nn'} + H_{nn'}^{11} \delta_{pp'} \\ &\quad + (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) V_{pn'n p'}^{phJ} + (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) V_{pn p'n'}^{ppJ} \\ B_{pn,p'n'}^J &= (-1)^{j_{p'} - j_{n'} + J} (u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) V_{pp'n n'}^{phJ} \\ &\quad - (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) V_{pn p'n'}^{ppJ}. \end{aligned}$$

However, the isovector part

$$B_{\lambda J}^\pm = \left| \sum_{pn} \langle p || T^J || n \rangle (X_{pn}^{\lambda J} u_p v_n + (-1)^J Y_{pn}^{\lambda J} v_p u_n) \right|^2.$$

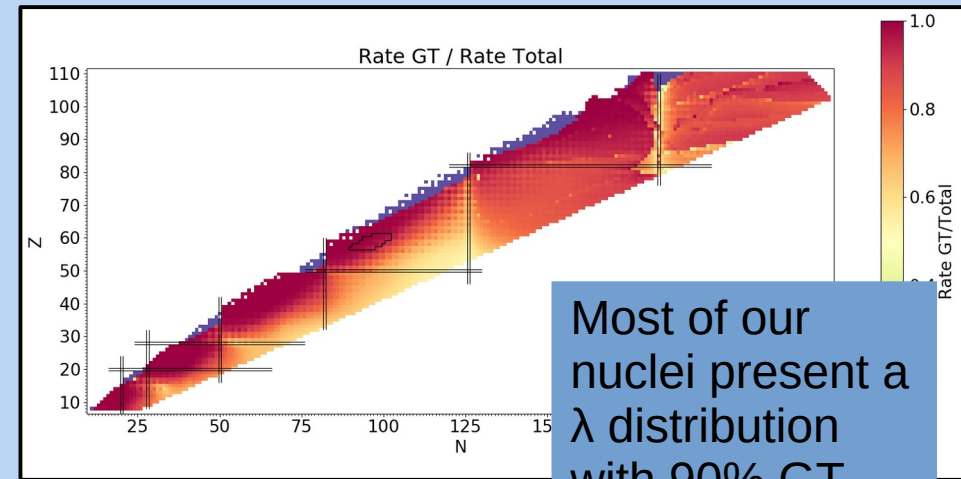
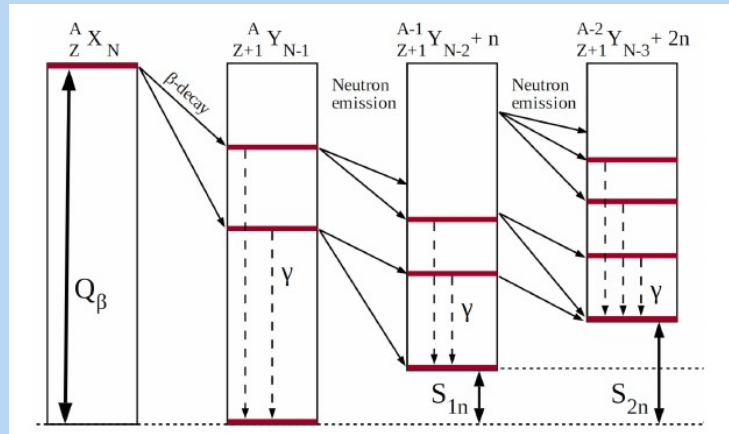
$$V_\tau^{T=0} = \mathbf{1}_1 \mathbf{1}_2$$

$$V_\tau^{T=1} = \vec{\tau}_1 \vec{\tau}_2 = 4\tau_{1z}\tau_{2z} + 2\tau_{1+}\tau_{2-} + 2\tau_{1-}\tau_{2+}$$

Talys Spin-Parity

We do not have a detailed description of these excited states but we can assume that all are populated by GT.

The other required information is the initial state from which the parent nucleus decayed. We can assume it was the ground state.



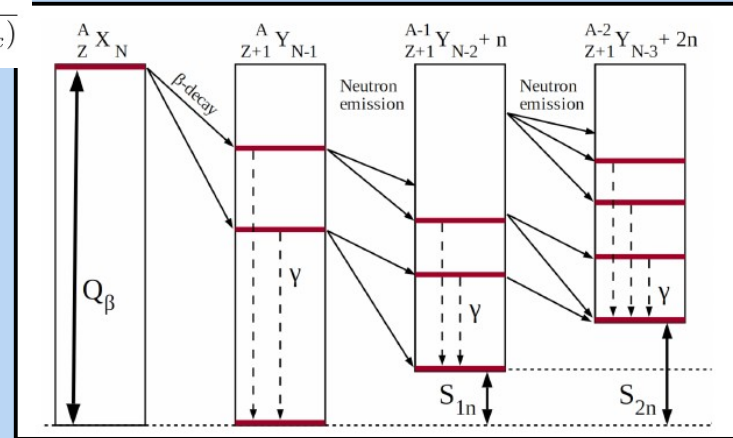
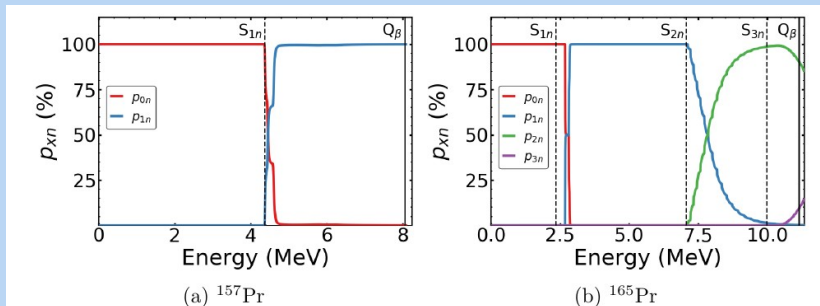
Hauser-Feshbach

This method describe the statistical decay of a compound nucleus.

It is the combination of the incident particle and the target creating a short-lived nucleus.

$$P_{xn} = \int_0^{Q_\beta} \lambda(E_x) \cdot p_{xn}(E_x) dE_x$$

$$p_{xn}(E_x) = \frac{\Gamma_{xn}(E_x)}{\Gamma_n(E_x) + \Gamma_\gamma(E_x)}$$



Inputs:

Level Densities
 γ -Strength function
 Optical model

Transmission
 coefficient

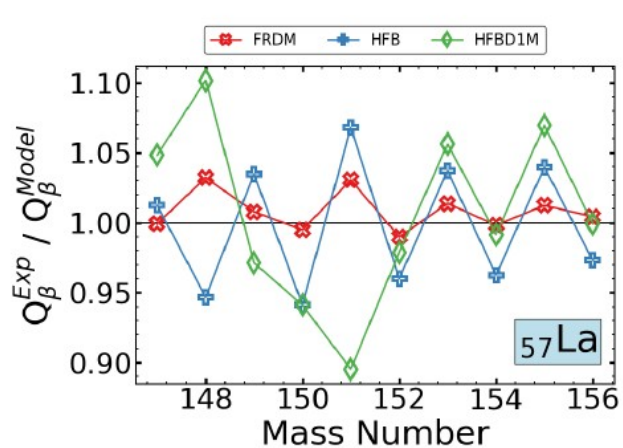
Mass models

- Skyrme-Hartree-Fock-Bogoliubov (HFB) [109]
- Gogny-Hartree-Fock-Bogoliubov (HFB1D) [110]
- Finite-Range Droplet Macroscopic (FRDM) [33]

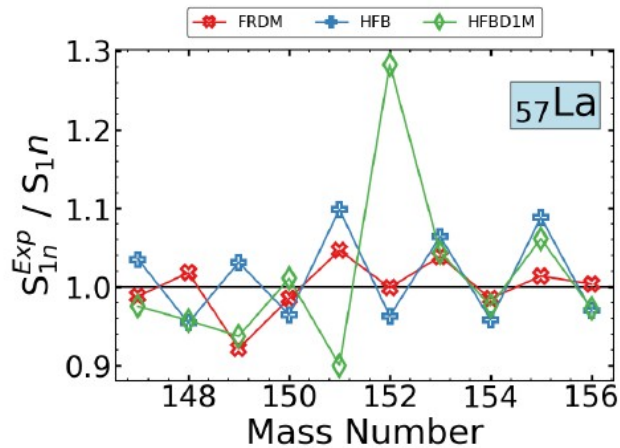
$$Q_{\beta^-} = M(Z, N) - M(Z + 1, N - 1)$$

$$S_{xn} = -M(Z + 1, N - 1) + M(Z + 1, N - x - 1) + xM_n$$

Model	Q_{β^-}	S_{1n}	Both
FRDM	0.039	0.055	0.034
HFB	0.042	0.058	0.041
HFBD1M	0.043	0.069	0.041

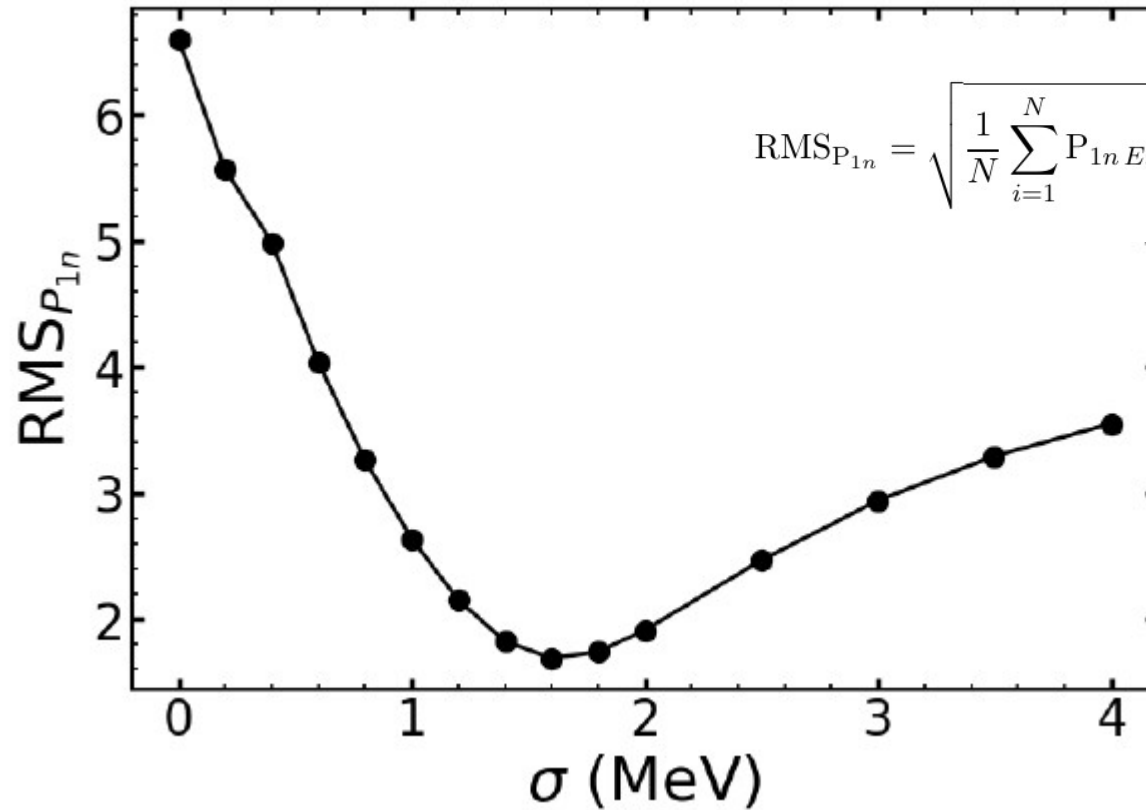


(a) Q_{β^-}



(b) S_{1n}

Sigma ajustment



$$RMS_{P_{1n}} = \sqrt{\frac{1}{N} \sum_{i=1}^N P_{1n \text{ Exp}} \left[\log_{10} \left(\frac{P_{1n \text{ Theory}}}{P_{1n \text{ Exp}}} \right)^2 \right]}$$